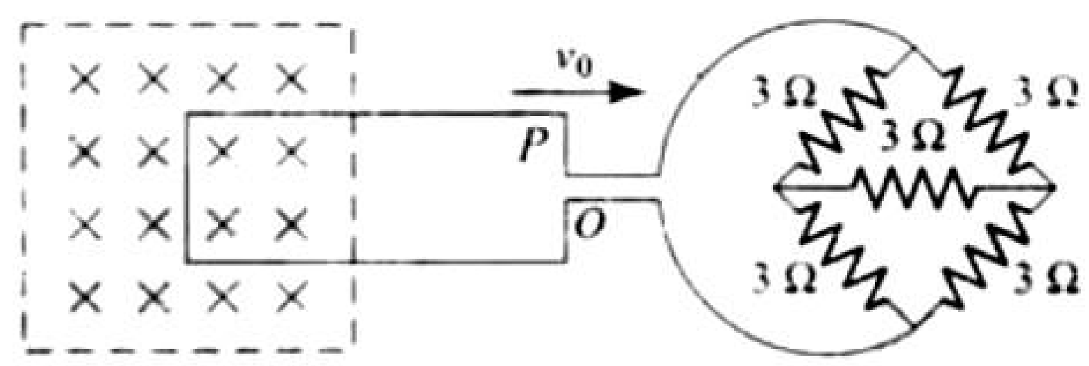
ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT [JEE ADVANCED PREVIOUS YEAR SOLVED PAPERS]

JEE Advanced

Single Correct Answer Type

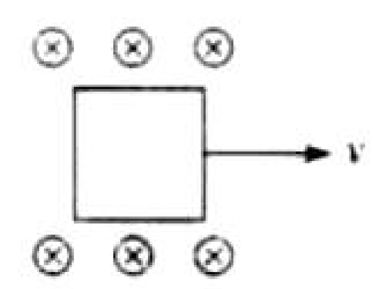
1. A square metal wire loop of side 10 cm and resistances 1 Ω is moved with constant velocity v_0 in a uniform magnetic field of induction $B = 2 \text{Wb m}^{-2}$, as shown in the figure. The magnetic field lines are perpendicular to the plane of loop and directed into the paper. The loop is connected to the network of resistances, each of value 3 Ω. The resistance of the lead wires is negligible. The speed of the loop so as to have a steady current of 1 mA in the loop is



a. 2 m s^{-1} **b.** 2 cm s^{-1} **c.** 10 m s^{-1} **d.** 20 m s^{-1}

(IIT-JEE 1983)

2. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to and into the plane of the loop exists everywhere.



The current induced in the loop is

a. BLv/R clockwise **b.** BLv/R anticlockwise

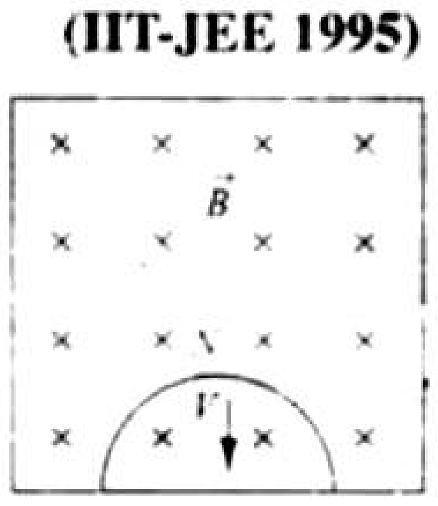
c. $2BLv/R$ anticlockwise **d.** zero **(IIT-JEE 1989)**

3. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

a. $\frac{BR}{A}$ **b.** $(AB)/R$ **c.** ABR **d.** $(E^2A)/R^2$

(IIT-JEE 1995)

4. A thin semicircular conducting ring of radius R is falling with its plane vertical in horizontal magnetic induction $\vec{B}$. At the position MNQ, the speed of the ring is V, and the potential difference developed across the ring is



a. zero

b. $BV\pi R^2/2$ and M is at higher potential

c. πRBV and Q is at higher potential

d. $2RBV$ and Q is at higher potential **(IIT-JEE 1996)**

5. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement(s) from the following:

a. The entire rod is at the same electric potential

b. There is an electric field in the rod

c. The electric potential is highest at the center of the rod and decrease toward its ends

d. The electric potential is lowest at the center of the rod and increases toward its ends **(IIT-JEE 1998)**

6. A small square loop of wire of side l is placed inside a large square loop of wire of side $L (L \gg l)$. The loops are coplanar and their centers coincide. The mutual inductance of the system is proportional to

a. l/L **b.** l^2/L **c.** L/l **d.** L^2/l

(IIT-JEE 1998)

7. A circular loop of radius R, carrying current I, lies in x–y plane with its center at origin. The total magnetic flux through x–y plane is

a. directly proportional to I

b. directly proportional to R

c. inversely proportional to R

d. zero **(IIT-JEE 1999)**

8. Two identical circular loops of metal wire are lying on a table without touching each other. Loop-A carries a current which increases with time. In response, the loop-B

a. remains stationary

b. is attracted by the loop A

c. is repelled by the loop A

d. rotates about its CM with CM fixed

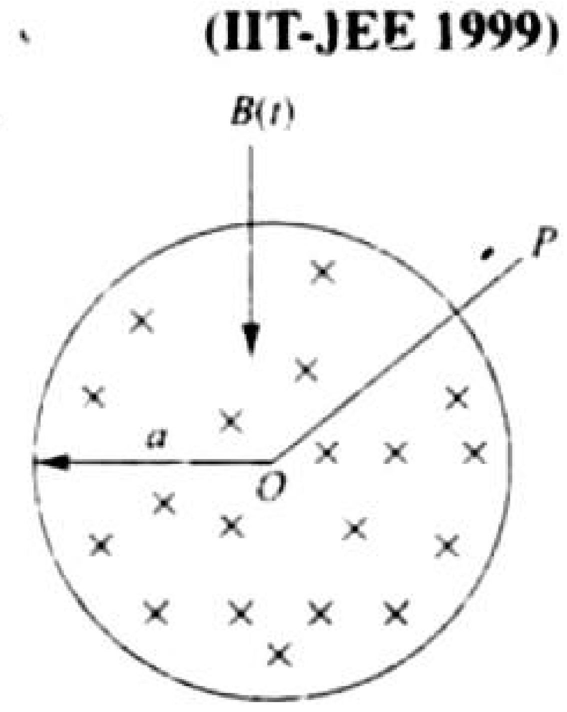
(IIT-JEE 1999)

9. A coil of inductance 8.4 mH and resistance 6 Ω is connected to a 12-V battery. The current in the coil is 1.0 A at approximately the time

a. 500 s **b.** 25 s **c.** 35 ms **d.** 1 ms

(IIT-JEE 1999)

10. A uniform but time-varying magnetic field $B(t)$ exists in a circular region of radius a and is directed into the plane of the paper as shown in the figure. The magnitude of the induced electric field at point P at a distance r from the center of the circular region



a. is zero **b.** decreases as $1/r$

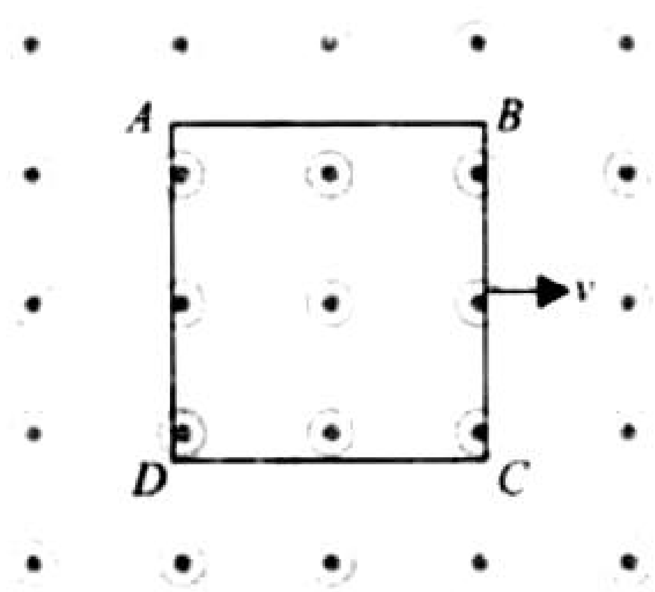
c. increases as r **d.** decreases as $1/r^2$

(IIT-JEE 2000)

11. A coil of wire having inductance and resistance has a conducting ring placed coaxially within it. The coil is connected to a battery at time $t = 0$, so that a time-dependent current $I_1(t)$ starts flowing through the coil. If $I_2(t)$ is the current induced in the ring, and $B(t)$ is the magnetic field at the axis of the coil due to $I_1(t)$, then as a function of time ($t > 0$), the product $I_2(t)B(t)$

a. increases with time
b. decreases with time
c. does not vary with time
d. passes through a maximum **(IIT-JEE 2000)**

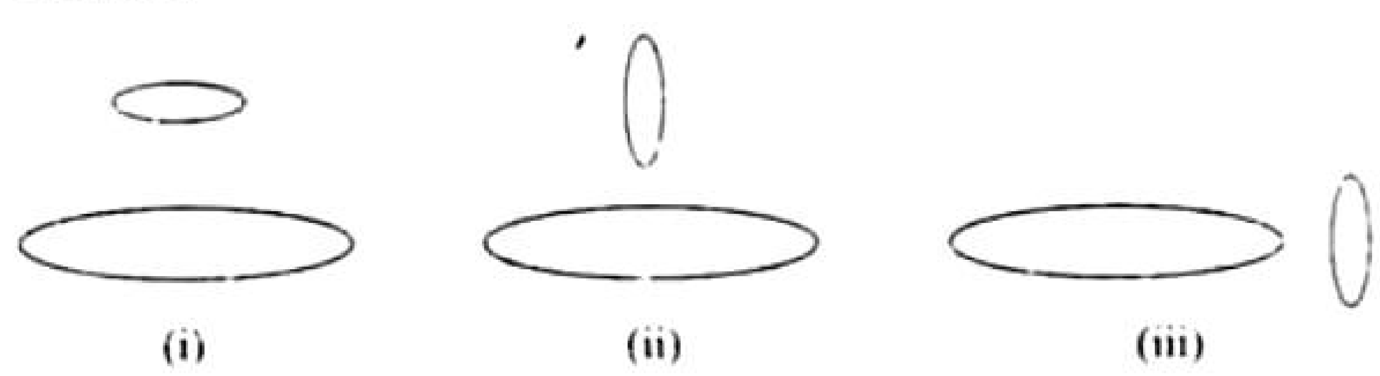
12. A metallic square loop $ABCD$ is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced



a. AD, but not in BC b. BC, but not in AD
c. neither AD nor in BC d. both AD and BC

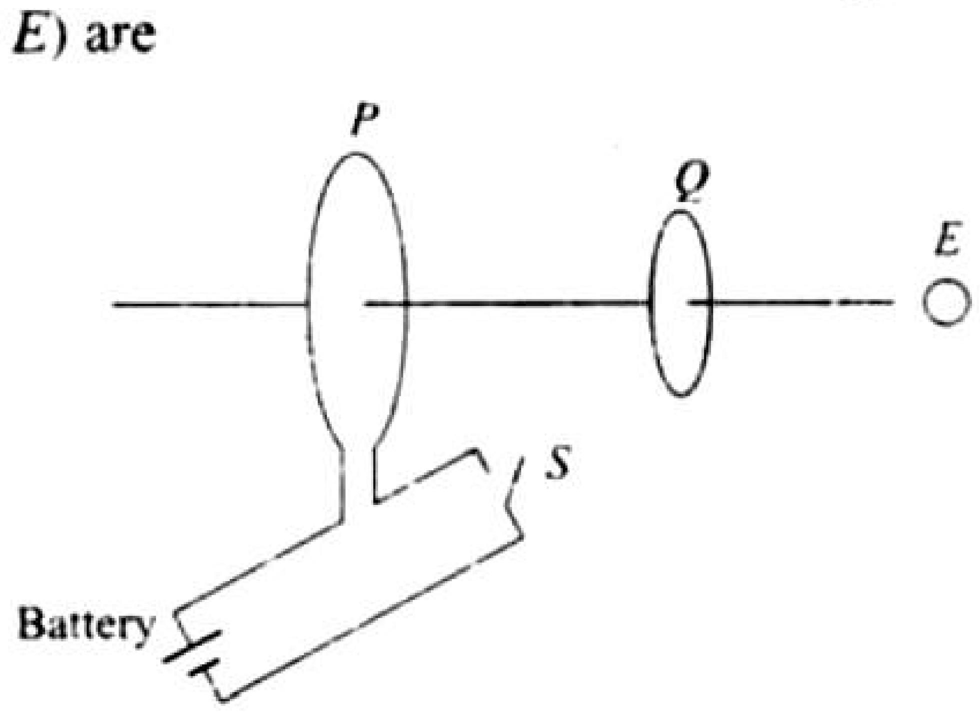
(IIT-JEE 2001)

13. Two circular coils can be arranged in any of the three situations shown in the figure. Their mutual inductance will be:



a. maximum in situation (i)
b. maximum in situation (ii)
c. maximum in situation (iii)
d. the same in all situations **(IIT-JEE 2001)**

14. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions in which I_{Q1} and I_{Q2} (as seen by E) are



a. respectively clockwise and anticlockwise
b. both clockwise
c. both anticlockwise
d. respectively anticlockwise and clockwise

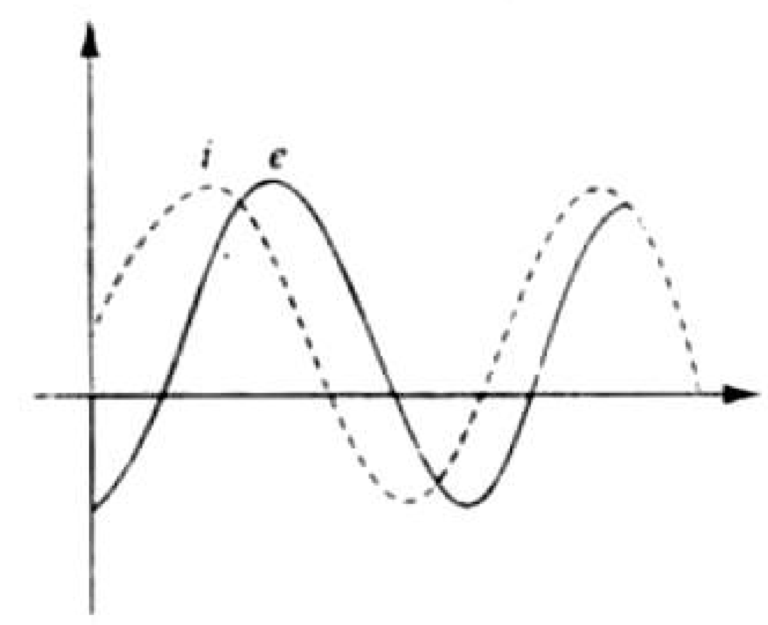
(IIT-JEE 2002)

15. A short-circuited coil is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be

a. halved b. the same
c. doubled d. quadrupled

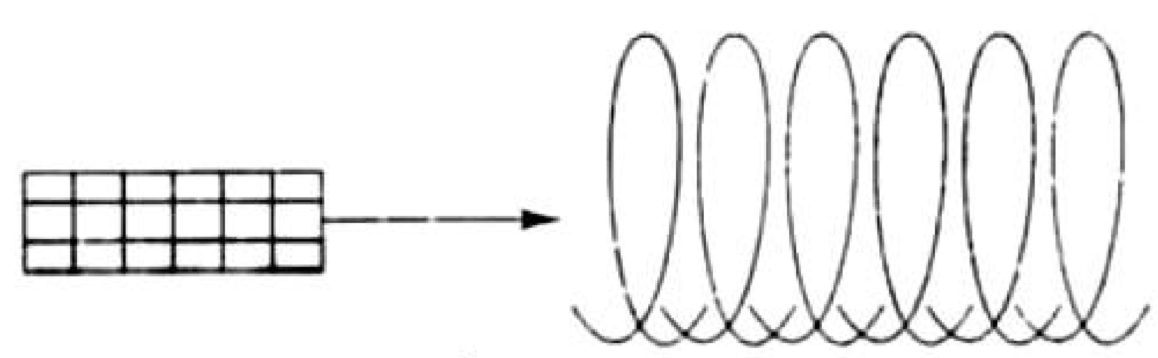
(IIT-JEE 2002)

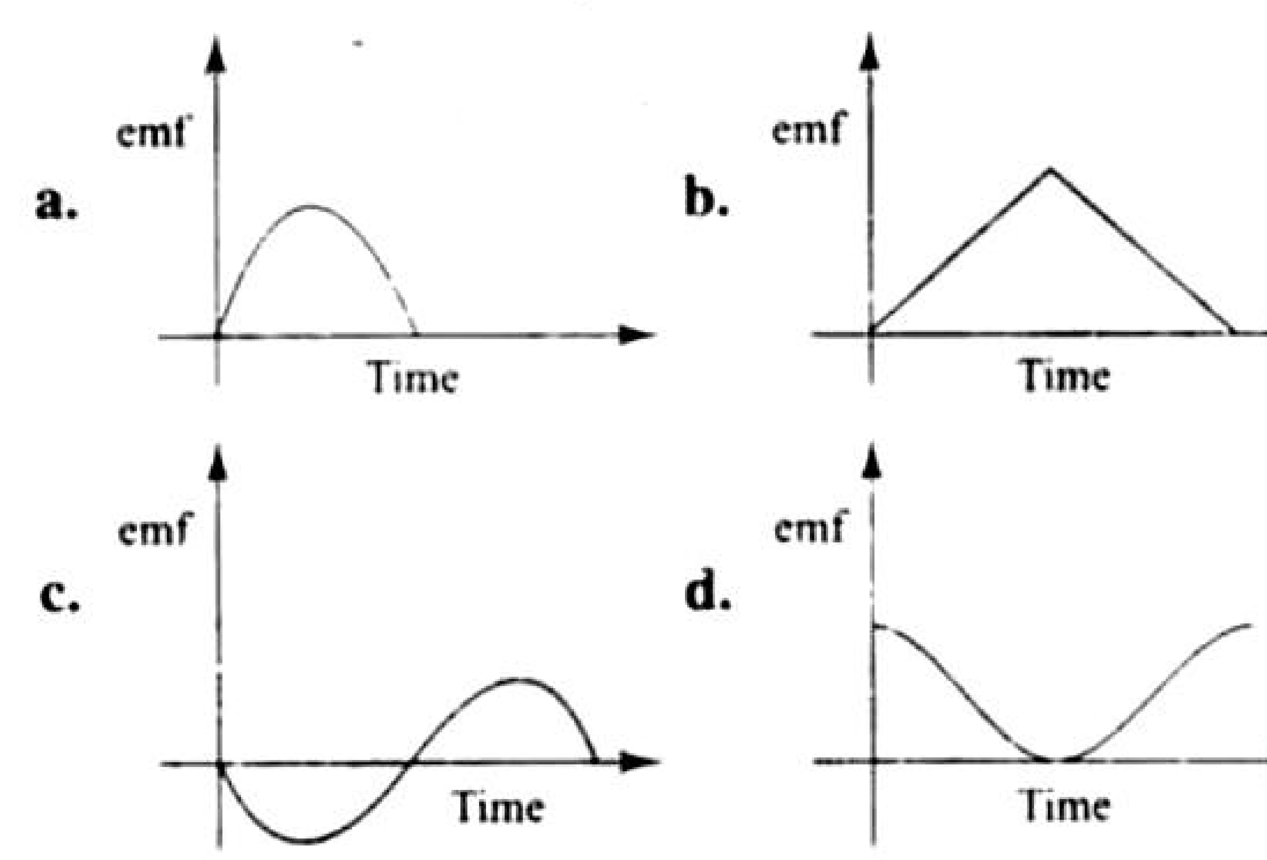
16. When an ac source of emf $e = E_0 \sin(100\,t)$ is connected across a circuit, the phase difference between emf e and current i in the circuit is observed to be $\pi/4$, as shown in the figure. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements.



a. $R = 1\ \text{k}\Omega$, $C = 10\ \mu\text{F}$
b. $R = 1\ \text{k}\Omega$, $C = 1\ \mu\text{F}$
c. $R = 1\ \text{k}\Omega$, $L = 10$ H
d. $R = 1\ \text{k}\Omega$, $L = 1$ H **(IIT-JEE 2003)**

17. A small bar magnet is being slowly inserted with constant velocity inside a solenoid as shown in the figure. Which graph best represents the relationship between emf induced with time?



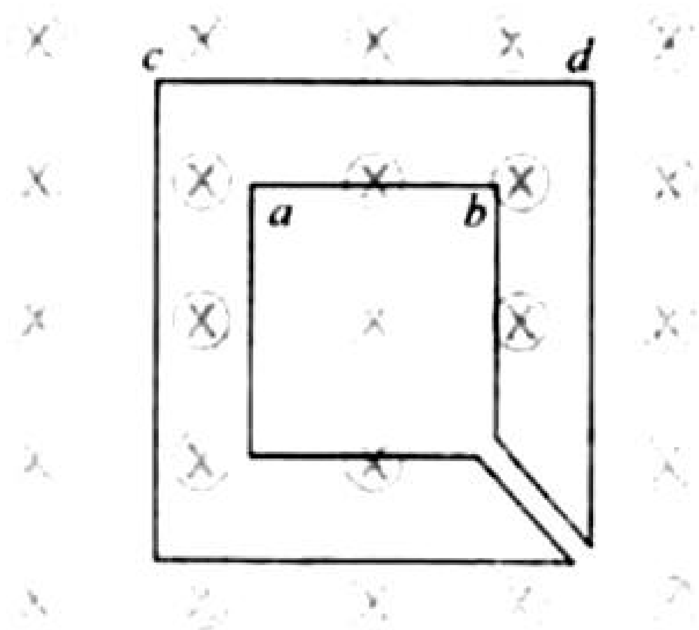


(IIT-JEE 2004)

18. An infinitely long cylinder is kept parallel to a uniform magnetic field B directed along positive z-axis. The direction of induced current as seen from the z-axis will be

a. zero
b. anticlockwise of the positive z-axis
c. clockwise of the positive z-axis
d. along the magnetic field **(IIT-JEE 2005)**

19. Figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time. I_1 and I_2 are the currents in the segments ab and cd. Then,



a. $I_1 > I_2$
b. $I_1 < I_2$
c. I_1 is in the direction ba and I_2 is in the direction cd
d. I_1 is in the direction ab and I_2 is in the direction dc

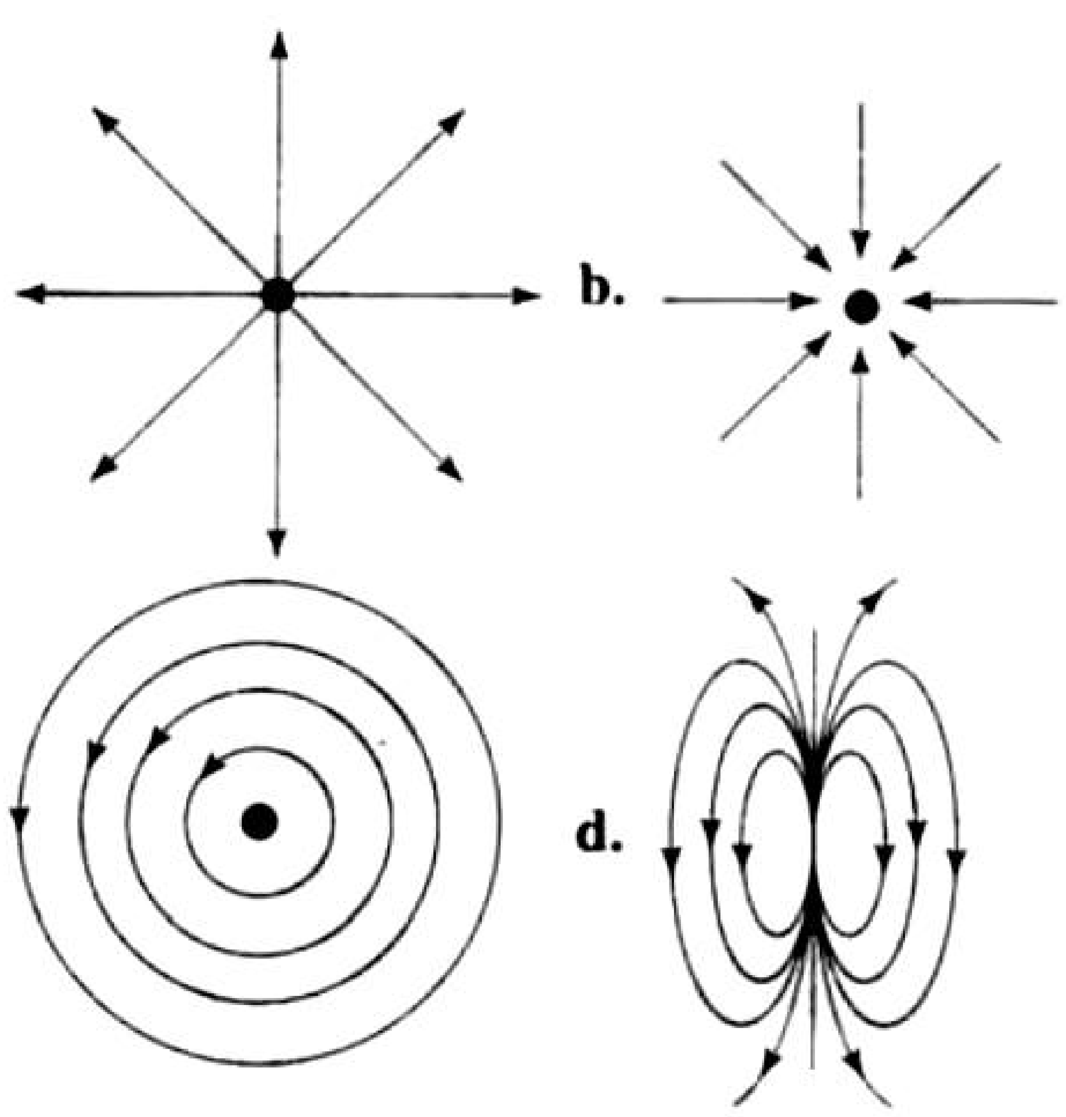
(IIT-JEE 2009)

20. An ac voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

a. the bulb glows dimmer
b. the bulb glows brighter
c. total impedance of the circuit is unchanged
d. total impedance of the circuit increases

(IIT-JEE 2010)

21. Which of the field patterns given below is valid for electric field as well as for magnetic field?

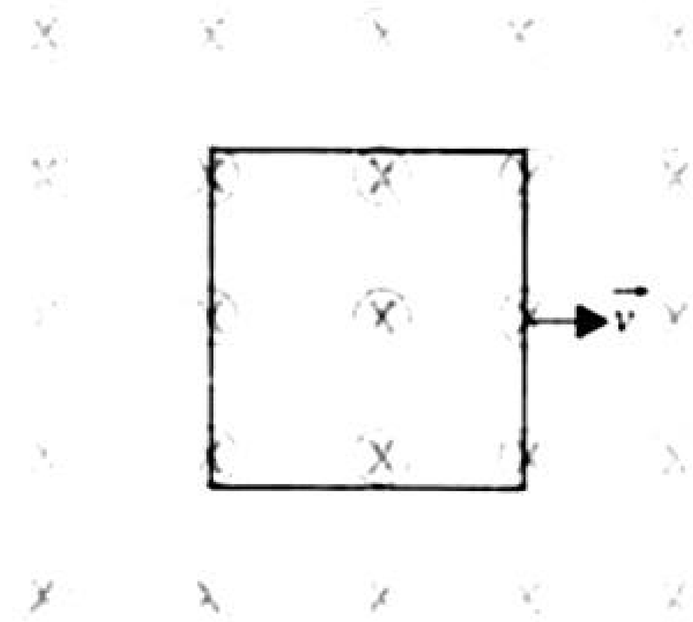


(IIT-JEE 2011)

Multiple Correct Answer Type

1. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere.

The current induced in the loop is



a. BLv/R clockwise
b. BLv/R anticlockwise
c. $2BLv/R$ anticlockwise
d. zero **(IIT-JEE 1989)**

2. Two different coils have self-inductances $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At that time, the current, the induced voltage and the energy stored in the first coil are i_1, V_1, and W_1, respectively. Corresponding values for the second coil at the same instant are i_1, V_2, and W_2, respectively. Then

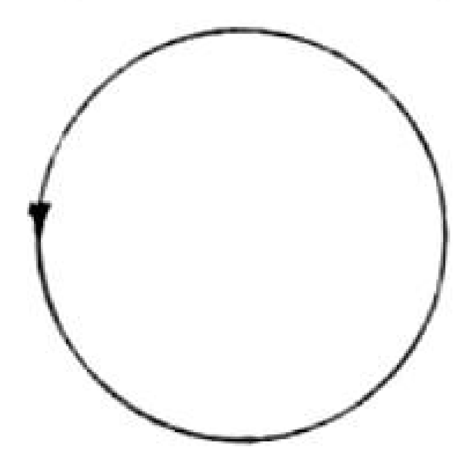
a. $\frac{i_1}{i_2} = \frac{1}{4}$
b. $\frac{i_1}{i_2} = 4$
c. $\frac{W_1}{W_2} = \frac{1}{4}$
d. $\frac{V_1}{V_2} = 4$ **(IIT-JEE 1994)**

3. The SI unit of inductance, the henry, can be written as

a. weber/ampere
b. volt-second/ampere
c. joule/(ampere)^2
d. ohm-second

(IIT-JEE 1998)

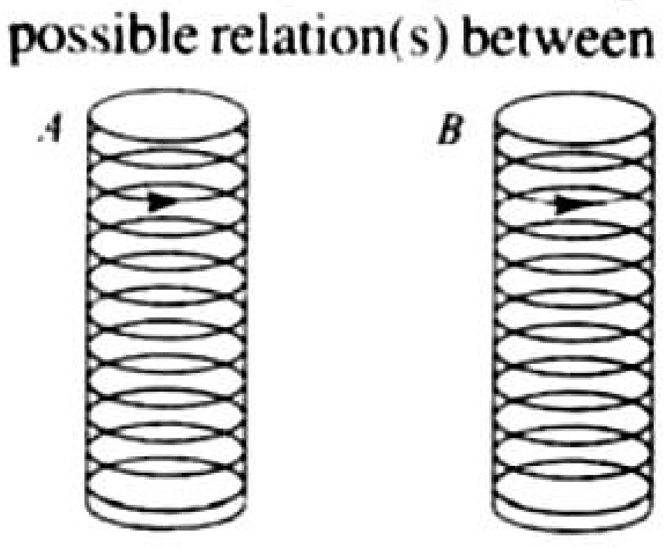
4. A field line is shown in the figure. This field cannot represent



a. Magnetic field
b. Electrostatic field
c. Induced electric field
d. Gravitational field

(IIT-JEE 2006)

5. Two metallic rings A and B, identical in shape and size but having different resistivities ρ_A and ρ_B, are kept on top of two identical solenoids as shown in the figure. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights h_A and h_B, respectively, with $h_A > h_B$. The possible relation(s) between their resistivities and their masses m_A and m_B is(are)



a. $\rho_A > \rho_B$ and $m_A = m_B$
b. $\rho_A < \rho_B$ and $m_A = m_B$
c. $\rho_A > \rho_B$ and $m_A > m_B$
d. $\rho_A < \rho_B$ and $m_A < m_B$

(IIT-JEE 2009)

6. A series R-C circuit is connected to an ac voltage source. Consider two cases: (A) when C is without a dielectric medium and (B) when C is filled with a dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

a. $I_R^A > I_R^B$ **b.** $I_R^A < I_R^B$

c. $V_C^A > V_C^B$ **d.** $V_C^A < V_C^B$

(IIT-JEE 2011)

7. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is/are

The current induced in the loop is

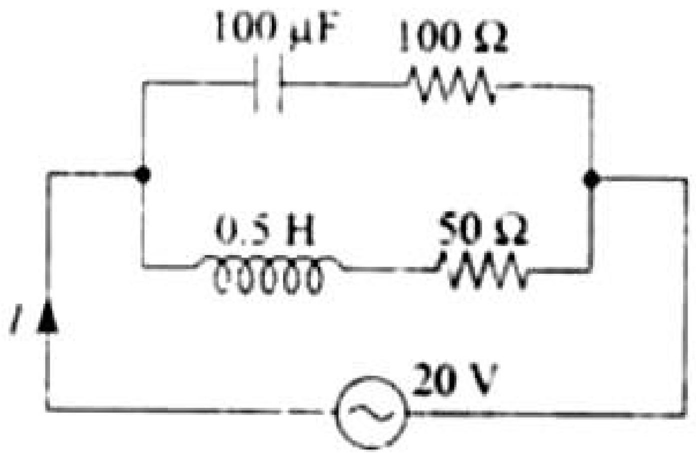
a. The emf induced in the loop is zero if the current is constant

b. The emf induced in the loop is finite if the current is constant

c. The emf induced in the loop is zero if the current decreases at a steady rate.

d. The emf induced in the loop is finite if the current decreases at a steady rate. **(IIT-JEE 2012)**

8. In the given circuit, the ac source has $\omega = 100 \text{ rad s}^{-1}$. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are



a. The current through the circuit, I is 0.3 A

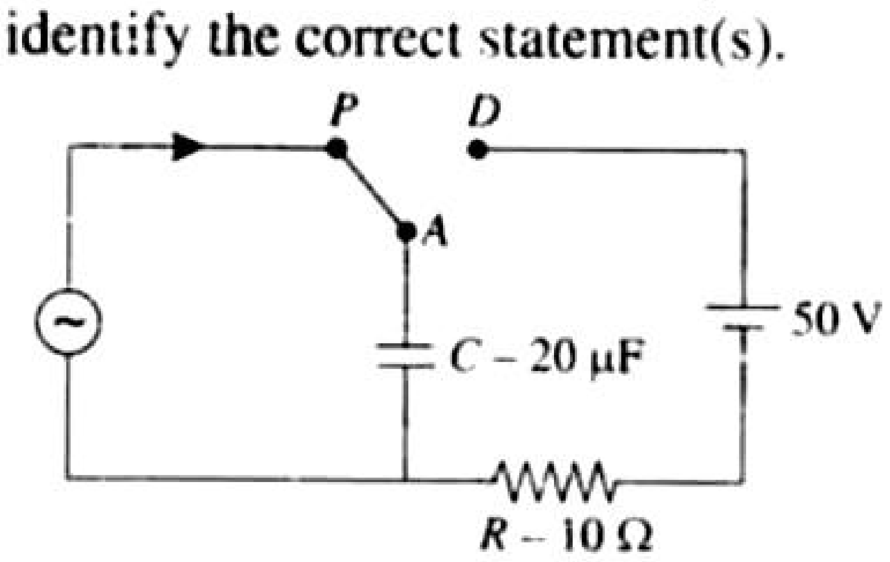
b. The current through the circuit, I is $0.3\sqrt{2}$ A

c. The voltage across 100 Ω resistor is $10\sqrt{2}$ V

d. The voltage across 50 Ω resistor is 10 V

(IIT-JEE 2012)

9. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1$ A and $\omega = 500$ rad/s starts flowing in it with the initial direction shown in the figure. At $t = 7\pi/6\omega$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu$F, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s).



a. Magnitude of the maximum charge on the capacitor before $t = \dfrac{7\pi}{6\omega}$ is 1×10^{-3}C

b. The current in the left part of the circuit just before $t = \dfrac{7\pi}{6\omega}$ is clockwise.

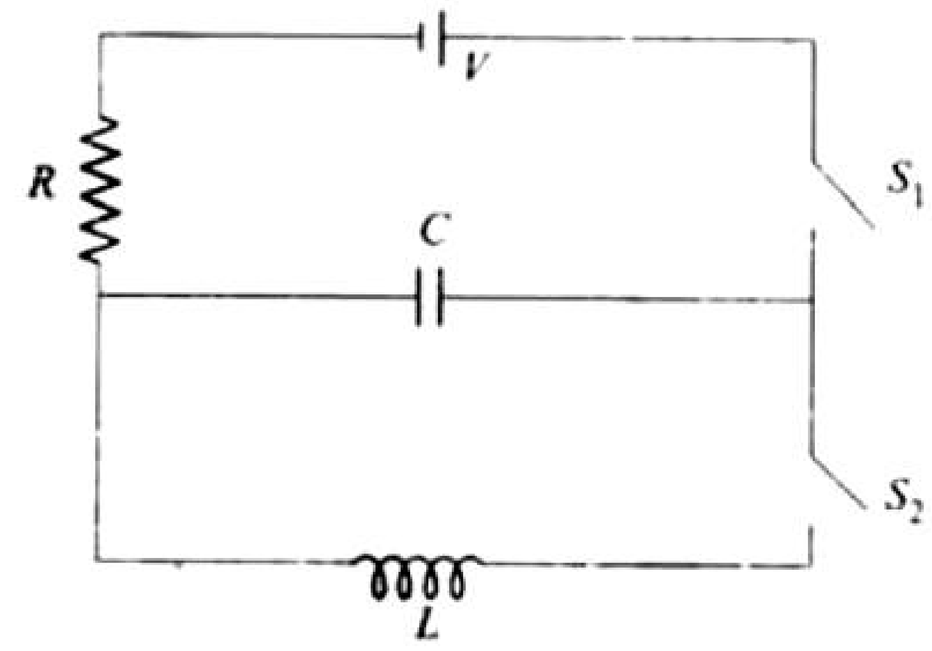
c. Immediately after A is connected to D, the current in R is 10 A.

d. $Q = 2 \times 10^{-3}$C. **(JEE Advanced 2014)**

Linked Comprehension Type

For Problems 1–3

In the given circuit shown in the figure, the capacitor (C) may be charged through resistance R by a battery V by closing switch S_1. Also, when S_1 is opened and S_2 is closed, the capacitor is connected in series with inductor (L). **(IIT-JEE 2006)**



1. At the start, the capacitor was uncharged. When switch S_1 is closed and S_2 is kept open, the time constant of this circuit is τ. Which of the following is correct?

a. After time interval τ, charge on the capacitor is $CV/2$

b. After time interval 2τ, charge on the capacitor of $CV(1 - e^{-2})$

c. The work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

d. After time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

2. When the capacitor gets charged completely, S_1 is opened and S_2 is closed. Then

a. at $t = 0$, energy stored in the circuit is purely in the form of magnetic energy.

b. at any time $t > 0$ current in the circuit is in the same direction.

c. at $t > 0$, there is no exchange of energy between the inductor and capacitor.

d. at any time $t > 0$, maximum instantaneous current in the circuit may be $V\sqrt{\dfrac{C}{L}}$.

3. Given that the total charge stored in the LC circuit is Q_0, the charge on the capacitor for $t \geq 0$ is

a. $Q = Q_0 \cos\left(\dfrac{\pi}{2} + \dfrac{t}{\sqrt{LC}}\right)$

b. $Q = Q_0 \cos\left(\dfrac{\pi}{2} - \dfrac{t}{\sqrt{LC}}\right)$

c. $Q = -LC\dfrac{d^2Q}{dt^2}$

d. $Q = -\dfrac{1}{\sqrt{LC}}\dfrac{d^2Q}{dt^2}$

For Problems 4–6

Modern trains are based on Maglev technology in which trains are magnetically levitated, which runs its EDS Maglev system.

There are coils on both sides of wheels. Due to motion of the train, current induces in the coil of track which levitate it. This is in accordance with Lenz's law. If train lowers down then due to Lenz's law, repulsive force increases due to which train gets uplifted and if it goes much higher then there is a net downward force due to gravity. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption and enabling the train to attain very high speeds.

Disadvantage of Maglev train is that as it slows down, the electromagnetic forces decrease and it becomes difficult to keep it levitated and as it moves forward according to Lenz's law there is an electromagnetic drag force. **(IIT-JEE 2006)**

4. What is the advantage of this system?
   a. mechanical force
   b. electrostatic attraction
   c. electrostatic repulsion
   d. magnetic repulsion
5. What is the disadvantage of this system?
   a. more friction
   b. less pollution
   c. less wear and tear
   d. high initial cost
6. Which force causes the train to elevate up?
   a. gravitational
   b. magnetic
   c. nuclear forces
   d. air drag

For Problems 7 and 8

Point Q is moving in a circular orbit of radius R in the x–y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $Q\omega/2\pi$. uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

(JEE Advanced 2013)

7. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change, is
   a. $\frac{BR}{4}$ b. $\frac{BR}{2}$ c. BR d. $2BR$
8. The change in the magnetic dipole moment associated with the orbit, at the end of time interval of the magnetic field change, is
   a. $-\gamma BQR^2$ b. $-\gamma \frac{BQR^2}{2}$
   c. $\gamma \frac{BQR^2}{2}$ d. γBQR^2

For Problems 9 and 10

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with the power factor unity. All the currents and voltage mentioned are rms values.

(JEE Advanced 2013)

9. If the direct transmission method with a cable of resistance 0.4 W km^{-1} is used, the power dissipation (in %) during transmission is
   a. 20 b. 30
   c. 40 d. 50
10. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1: 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is
    a. 200 : 1 b. 150 : 1
    c. 100 : 1 d. 50 : 1

Matching Column Type

1. Some law/processes are given in Column I. Match these with the physical phenomena given in Column II:

	Column I		Column II
i.	Dielectric ring uniformly charged	a.	Time independent electrostatic field out of system
ii.	Dielectric ring uniformly charged, rotating with angular velocity ω	b.	Magnetic field
iii.	Constant current in ring i_0	c.	Induced electric field
iv.	$i = i_0 \cos \omega t$	d.	Magnetic moment

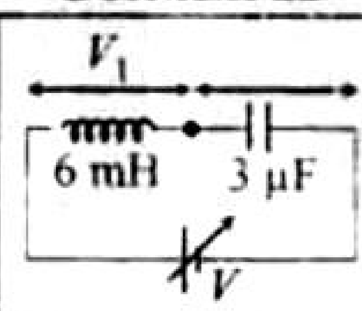
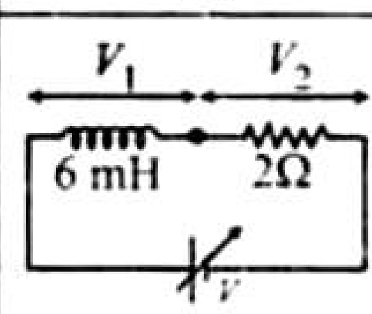
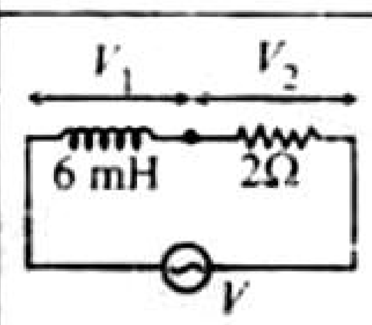
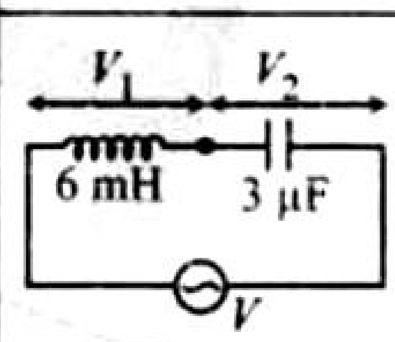
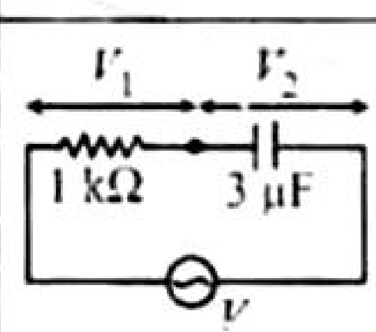
(IIT-JEE 2006)

2. Column I gives certain situations in which a straight metallic wire of resistance R is used and Column II gives some resulting effects. Match the statements in Column I with the statements in Column II:

	Column I		Column II
i.	A charged capacitor is connected to the ends of the wire	**a.**	A constant current flows through the wire
ii.	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	**b.**	Thermal energy is generated in the wire
iii.	The wire is placed in a constant electric field that has a direction along the length of the wire	**c.**	A constant potential difference develops between the ends of the wire
iv.	A battery of constant emf is connected to the ends of the wire	**d.**	Charges of constant magnitude appear at the ends of the wire

(IIT-JEE 2007)

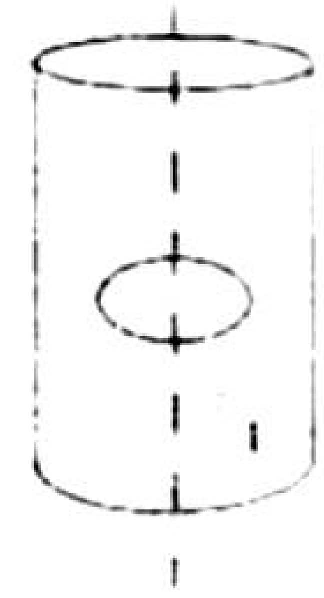
3. You are given many resistors, capacitors and inductors. These are connected to a variable dc voltage source (the first two circuits) or an ac voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for dc or rms for ac) flows through the circuit, the corresponding voltages V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the two.

	Column I		Column II
i.	$I \neq 0$, V_1 is proportional to I	**a.**	
ii.	$I \neq 0$, $V_2 > V_1$	**b.**	
iii.	$V_1 = 0$, $V_2 = V$	**c.**	
iv.	$I \neq 0$, V_2 is proportional to I	**d.**	
		e.	

(IIT-JEE 2010)

Integer Answer Type

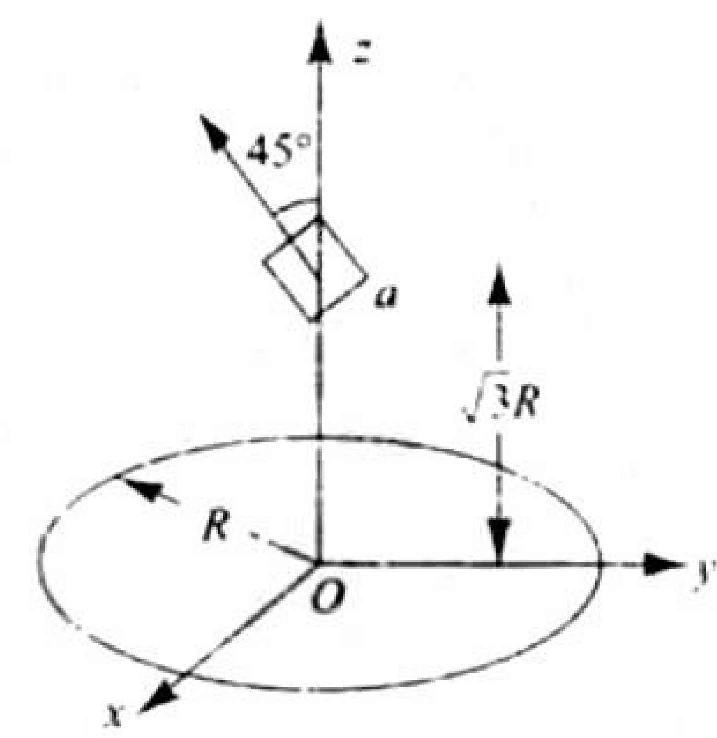
1. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown in the figure. A wire-loop of resistance 0.005 Ω and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos(300\,t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then N is



(IIT-JEE 2011)

2. A series R-C combination is connected to an ac voltage of angular frequency $\omega = 500$ rad s^{-1}. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **(IIT-JEE 2011)**

3. A circular wire loop of radius R is placed in the x–y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at along the axis of the circular wire loop as shown in the figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is **(IIT-JEE 2012)**



Assertion-Reasoning Type

Mark your answer as

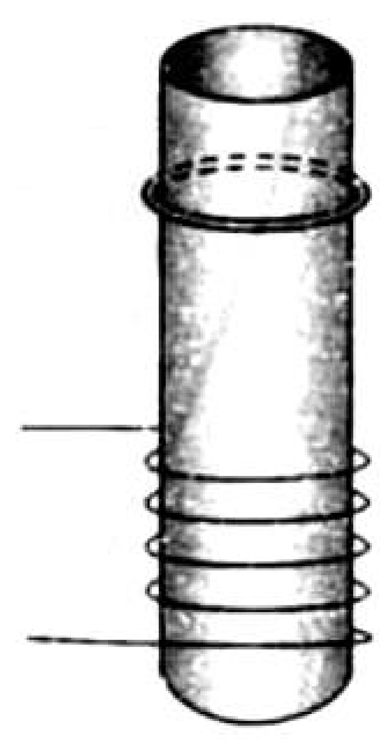
a. Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I.

b. Statement I is true, Statement II is true; Statement II is NOT the correct explanation for Statement I.

c. Statement I is true, Statement II is false.

d. Statement I is false, Statement II is true.

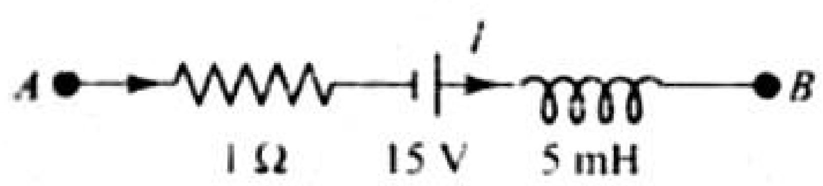
1. **Statement I:** A vertical iron rod has coil of wire wound over it at the bottom end. An alternating current flows in the coil. The rod goes through a conducting ring as shown in the figure. The ring can float at a certain height above the coil.



Statement II: In the above situation, a current is induced in the ring which interacts with the horizontal component of the magnetic field to produce an average force in the upward direction. **(IIT-JEE 2007)**

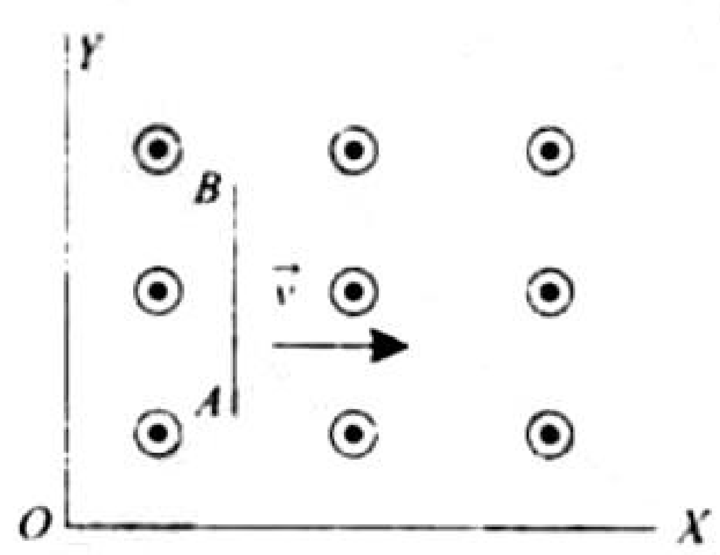
Fill in the Blanks Type

1. A uniformly wound solenoidal coil of self-inductance 1.8×10^{-4} H and resistance 6 Ω is broken up into two identical coils. These identical coils are then connected in parallel across a 15 V battery of negligible resistance. The time constant for the current in the circuit is ______ seconds and the steady state current through the battery is ______ amperes. **(IIT-JEE 1989)**
2. If ε_0 and μ_0, respectively, are the electric permittivity and magnetic permeability of free space, ε and μ are corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameter is ______. **(IIT-JEE 1992)**
3. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will be ______. **(IIT-JEE 1993)**
4. The network shown in the figure is part of a complete circuit. If at a certain instant the current (I) is 5 A, and is decreasing at a rate of 10^5 A s^{-1} then $V_B - V_A =$ –V. **(IIT-JEE 1997)**



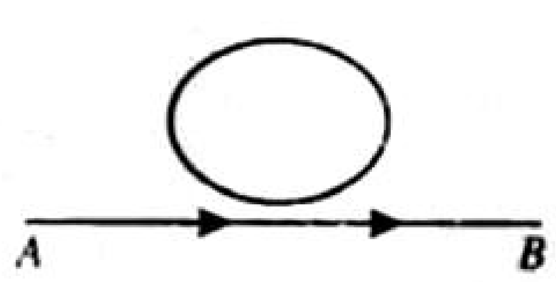
True/False Type

1. A coil of metal wire is kept stationary in a non-uniform magnetic field. An emf is induced in the coil. **(IIT-JEE 1986)**
2. A conducting rod AB moves parallel to the x-axis in a uniform magnetic field pointing in the positive z direction. The end A of the rod gets positively charged. **(IIT-JEE 1987)**

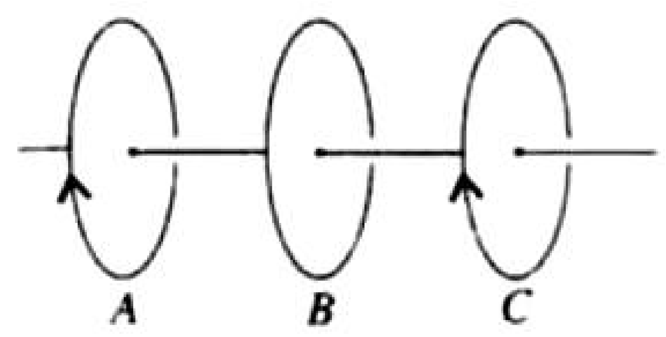


Subjective Type

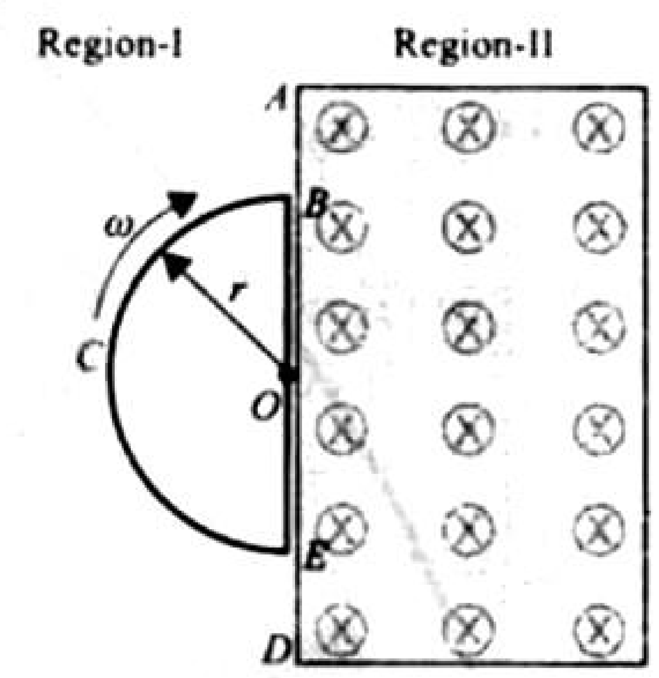
1. A current from A to B is increasing in magnitude. What is the direction of induced current, in the loop as shown in the figure? **(IIT-JEE 1981)**



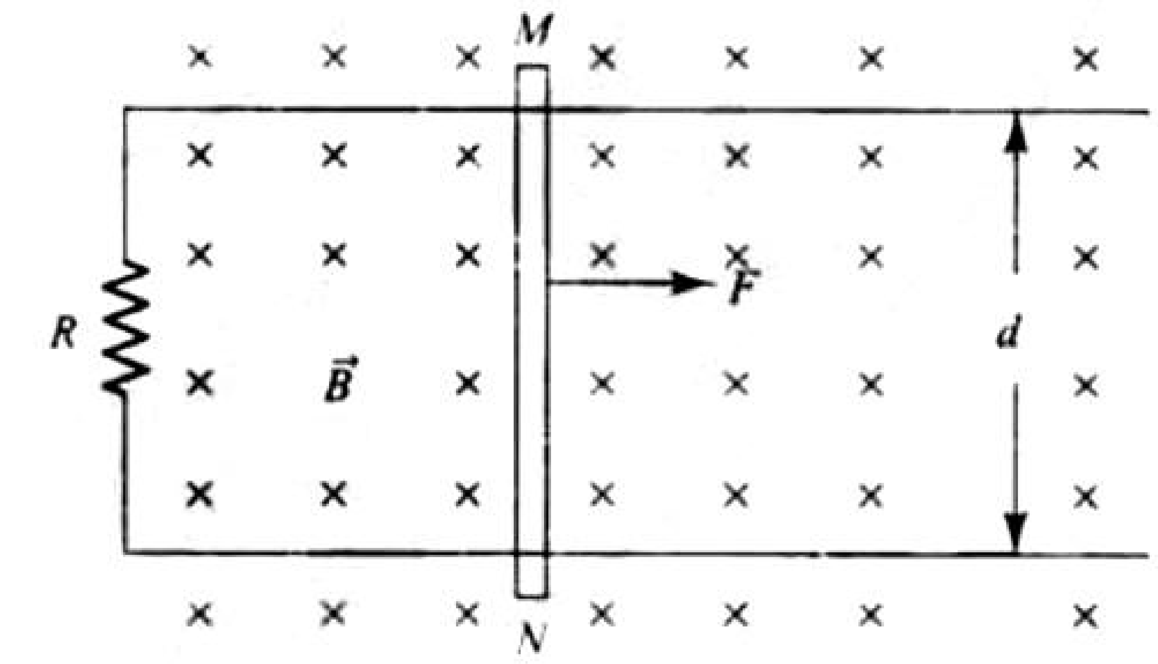
2. The two rails of a railway track, insulated from each other and the ground, are connected to a millivolt meter. What is the reading of the milli voltmeter when a train travels at a speed of 180 km/hour along the track, given that the horizontal components of earth's magnetic field is 0.2×10^{-4} webers/m^2 and the rails are separated by 1 meter? **(IIT-JEE 1981)**
3. Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in the figure. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is they any induced current in B? If no, give reason. If yes, mark the direction of the induced current in the diagram. **(IIT-JEE 1982)**



4. Space is divided by the line AD into two regions. Region I is field free and region II has a uniform magnetic field B directed into the plane of the paper. BCE is a semicircular conducting loop of radius r with center at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity ω about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R.

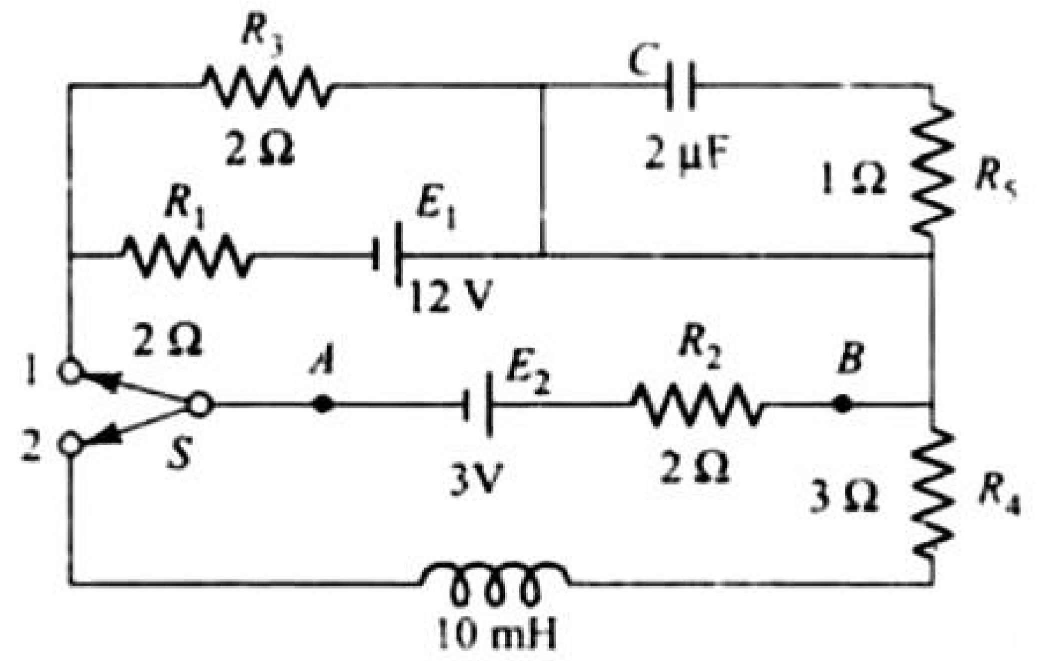


   a. Obtain an expression for the magnitude of the induced current in the loop.
   b. Show the direction of the current when the loop is entering into region II.
   c. Plot a graph between the induced emf and the time of rotation for the two periods of rotation. **(IIT-JEE 1985)**
5. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see Figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper. A variable force F is applied to rod MN such that as the rod moves, a constant current flows through R.



   a. Find the velocity of the rod and the applied force F as a function of the distance x of the rod from R.
   b. What fraction of the work done per second by F is converted into heat? **(IIT-JEE 1988)**

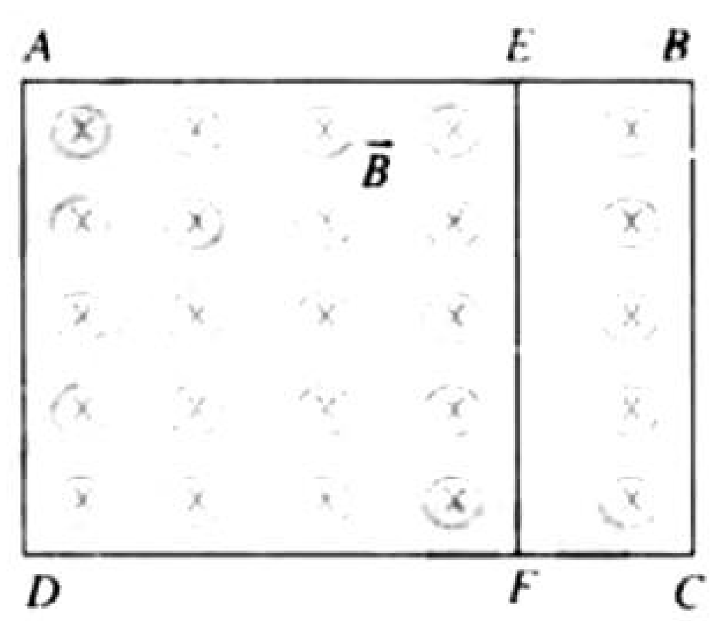
6. A circuit containing a two position switch S is shown in the below figure.



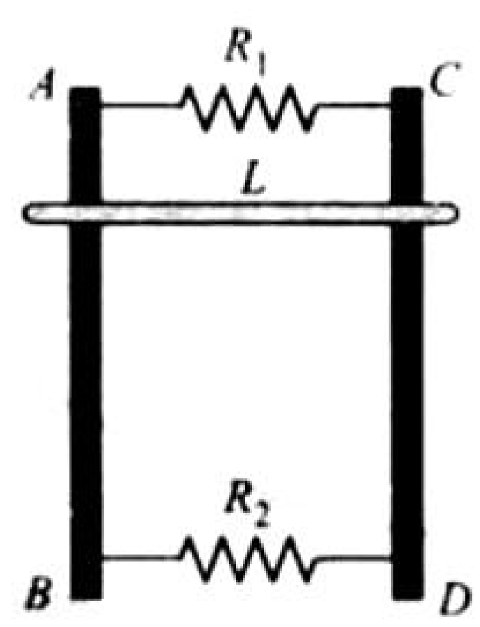
a. Switch S is in position 1. Find the potential difference $V_A - V_B$ and the rate of production of heat in R_1.

b. If now switch S is put in position 2 at $t = 0$, find the steady current in R_4 and the time when current in R_4 is half the steady value. Also calculate the energy stored in inductor L at that time. **(IIT-JEE 1991)**

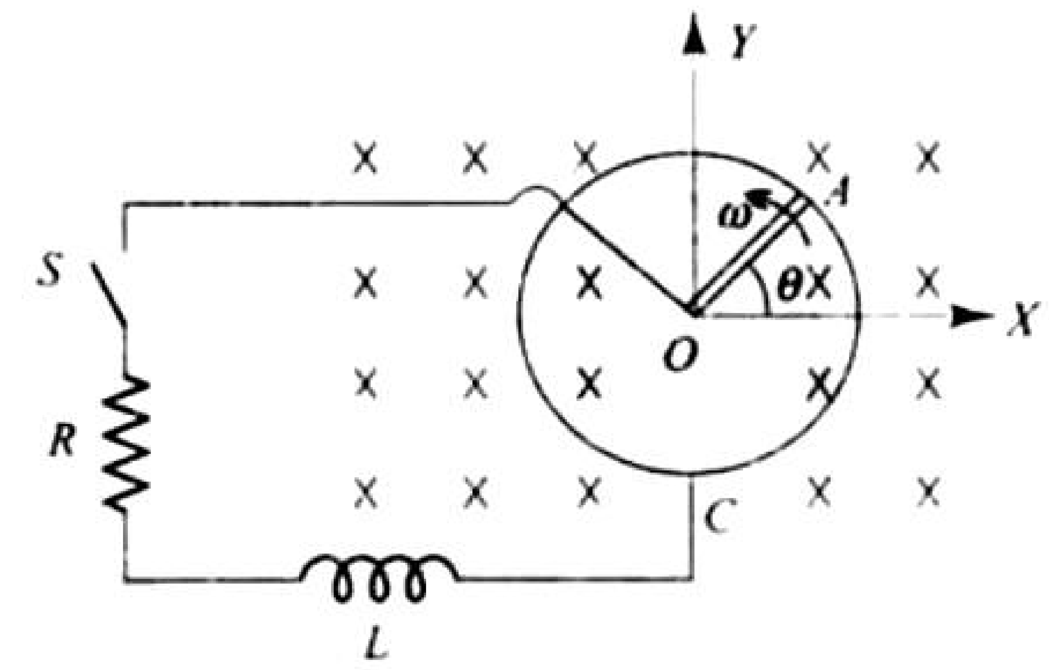
7. A rectangular frame $ABCD$, made of a uniform metal wire, has a straight connection between E and F made of the same wire, as shown in the figure. $AEFD$ is a square of side 1 m, and $EB = FC = 0.5$ m. The entire circuit is placed in steady increasing, uniform magnetic field directed into the plane of the paper and normal to it. The rate of change of the magnetic field is 1 T/s. The resistance per unit length of the wire is 1 Ω/m. Find the magnitudes and directions of the currents in the segments AE, BE and EF. **(IIT-JEE 1993)**



8. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in the figure. A horizontal metallic bar L of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 and 1.2 W, respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. **(IIT-JEE 1994)**



9. A metal rod OA of mass m and length r is kept rotating with a constant angular speed in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction is applied perpendicular and into the plane of rotation as shown in the figure. An inductor L and an external resistance R are connected through a switch S between point O and point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.



a. What is the induced emf across the terminals of the switch?

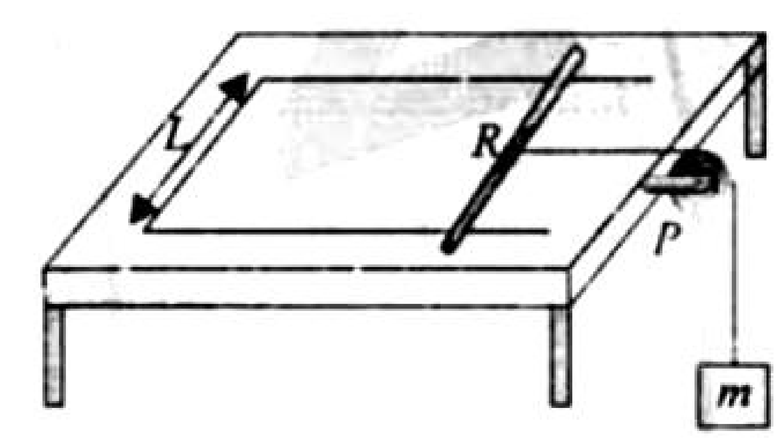
b. Switch S is closed at time $t = 0$.

i. Obtain an expression for the current as a function of time.

ii. In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that rod OA was along the positive X-axis at $t = 0$. **(IIT-JEE 1995)**

10. A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? **(IIT-JEE 1996)**

11. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table if the system is released from rest. Calculate



a. the terminal velocity achieved by the rod, and

b. the acceleration of the mass at the instant when the velocity of the rod is half the terminal velocity. **(IIT-JEE 1997)**

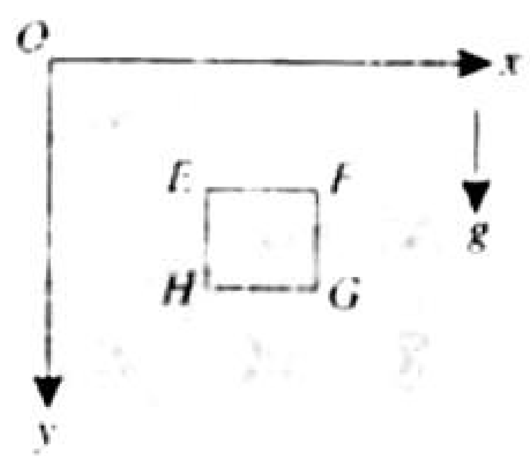
12. An infinitely small bar magnetic of dipole moment M is pointing and moving with the speed n in the X-direction A small closed circular conducting loop of radius a and negligible self-inductance lies in the Y-Z plane with its centre of $x = 0$ and its axis coinciding with the X-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a. **(IIT-JEE 1997)**

13. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting LC circuit is set oscillating at its natural frequency. Let

Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of charge Q is 200 μC.

a. When $Q = 100$ μC, what is the value of $\left|\frac{dI}{dt}\right|$?
b. When $Q = 200$ μC, what is the value of I?
c. Find the maximum value of I.
d. When I is equal to one-half its maximum value, what is the value of $|Q|$? **(IIT-JEE 1998)**

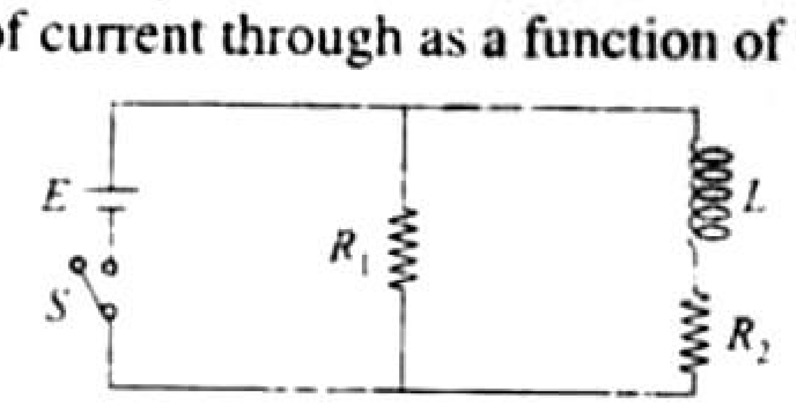
14. A magnetic field $B = B_0(y/a)\hat{k}$ is into the paper in the $+z$ direction. B_0 and a are positive constants. A square loop $EFGH$ of side a, mass m and resistance R, in x-y plane, starts falling under the influence of gravity see figure). Note the direction of x and y axes in figure. Find



a. the induced current in the loop and indicate its direction.
b. the total Lorentz force acting on the loop and indicate its direction, and
c. an expression for the speed of the loop, $v(t)$ and its terminal value. **(IIT-JEE 1999)**

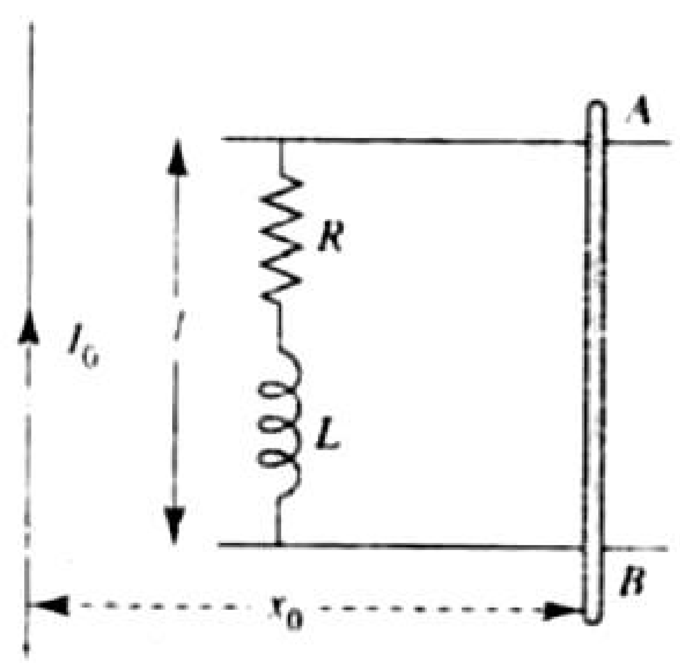
15. A thermocol vessel contains 0.5 kg of distilled water at 30°C. A metal coil of area 5×10^{-3} m^2, number of turns 100, mass 0.06 kg and resistance 1.6 Ω is lying horizontally at the bottom of the vessel. A uniform time-varying magnetic field is set up to pass vertically through the coil at time $t = 0$. The field is first increased from zero to 0.8 T at a constant rate between 0 and 0.2 s and then decreased to zero at the same rate between 0.2 and 0.4 s. The cycle is repeated 12000 times. Make sketches of the current through the coil and the power dissipated in the coil as function of time for the first two cycles. Clearly indicate the magnitude of the quantities on the axes. Assume that no heat is lost to the vessel or the surroundings. Determine the final temperature of water under thermal equilibrium. Specific heat of metal = 500 J $kg^{-1}K^{-1}$ and the specific heat of water = 4200 J $kg^{-1}K^{-1}$. Neglect the inductance of coil. **(IIT-JEE 2000)**

16. An inductor of inductance L = 400 mH and resistors of resistance $R_1 = 4$ Ω and $R_2 = 2$ Ω are connected to a battery of emf $E = 12$ V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time $t = 0$. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through as a function of time?



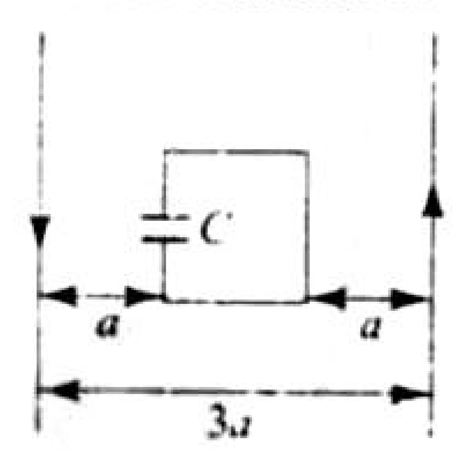
(IIT-JEE 2001)

17. A metal bar AB can slide on two parallel thick metallic rails separated by a distance l. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of rails and perpendicular to them as shown. The bar AB is made to slide on the rails away from the wire. Answer the following questions:

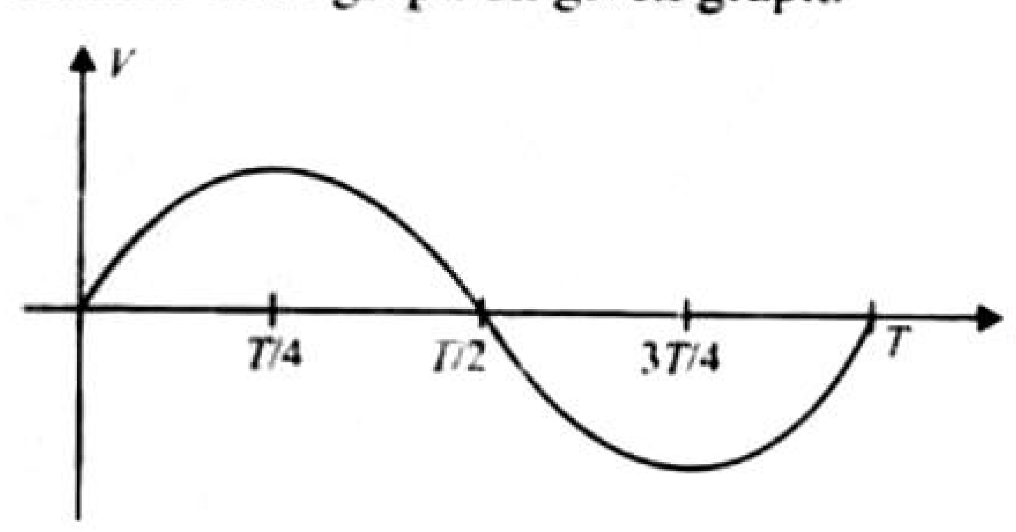


a. Find a relation among i, di/dt and $d\phi/dt$, where i is the current in the curcuit and ϕ is the flux of the magnetic field due to the long wire through the circuit.
b. It is observed that at time $t = T$, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i_1. Obtain an expression for the net charge that has flown through resistance R from $t = 0$ to $t = T$.
c. The bar is suddenly stopped at time T. The current through resistance R is found to be $i_1/4$ at time $2T$. Find the value of L/R in terms of the other given quantities. **(IIT-JEE 2002)**

18. Two infinitely long parallel wires carrying currents $I = I_0 \sin \omega t$ in opposite direction are placed at a distance $3a$ apart. A square loop of side a of negligible resistance with a capacitor of capacitance C is placed in the plane of wires as shown in the figure. Find the maximum current in the square loop. Also sketch the graph showing the variation of charge on the upper plate of the capacitor as a function of time for one complete cycle taking anticlockwise direction for the current in the loop as positive. **(IIT-JEE 2003)**

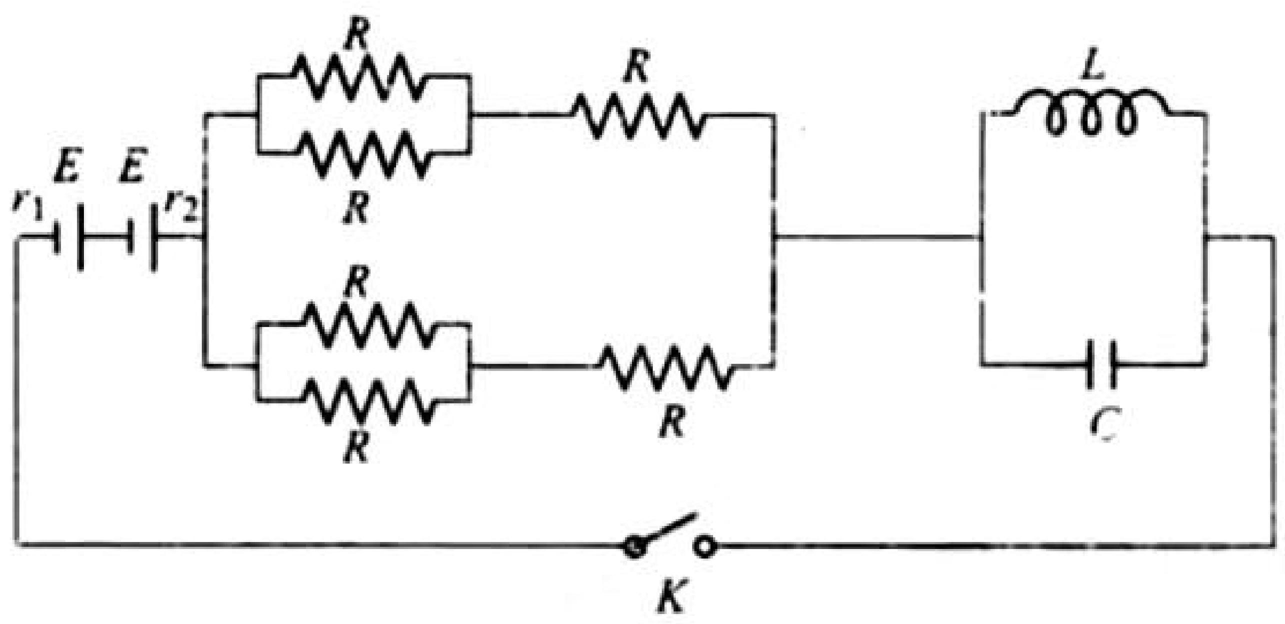


19. In a series L-R circuit (L = 35 mH and R = 11 Ω), a variable emf source ($V = V_0 \sin \omega t$) of V_{rms} = 220 V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current–time graph on given graph.



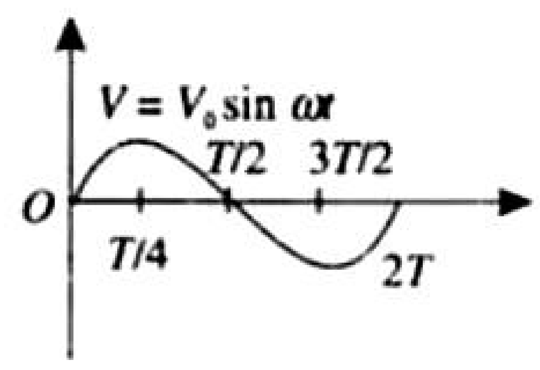
(IIT-JEE 2004)

20. In a circuit shown in the figure, A and B are two cells of same emf E but different internal resistances r_1 and r_2 ($r_1 > r_2$), respectively. Find the value of R such that the potential difference across the terminals of cell A is zero along time after key K is closed.

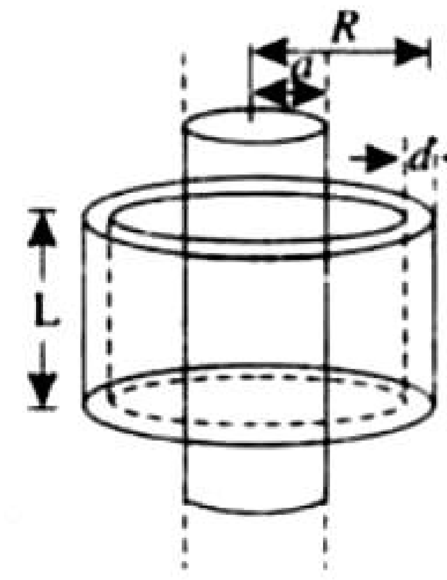


(IIT-JEE 2004)

21. In a series L-R circuit ($L = 35$ mH and $R = 11\ \Omega$), a variable emf source ($V = V_o \sin \omega t$) of $V_{rms} = 220$ V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current– time graph on given graph $\left(\pi = \frac{22}{7}\right)$. **(IIT-JEE 2004)**



22. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d ($d << R$) and length L. A variable current flows through the coil. If the resistivity of the material of cylindrical shell is, find the induced current in the shell. **(IIT-JEE 2005)**



ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. b. **2.** d. **3.** b. **4.** d. **5.** b.
6. b. **7.** d. **8.** c. **9.** d. **10.** b.
11. d. **12.** d. **13.** a. **14.** d. **15.** b.
16. a. **17.** c. **18.** a. **19.** d. **20.** b.
21. c.

Multiple Correct Answers Type

1. d. **2.** a., c., d. **3.** a., b., c., d.
4. b., d. **5.** b., d. **6.** b., c. **7.** a., c.
8. a., c. **9.** c., d

Linked Comprehension Type

1. b. **2.** d. **3.** c. **4.** a **5.** d.
6. b. **7.** b. **8.** b. **9.** b **10.** a.

Matching Column Type

1. i.→a.; ii.→a., b., d.; iii.→b., d.; iv.→b., c.
2. i.→b.; ii.→c., d.; iii.→d.; iv.→a., b., c.
3. i.→c., d., e.; ii.→b., c., d., e.; iii.→a., b.; iv.→b., c., d., e.

Integer Answer Type

1. (6) **2.** (4) **3.** (7)

Assertion–Reasoning Type

1. a.

Fill in the Blanks Type

1. 8 A **2.** $\sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}}$ **3.** Left to right or zero
4. 15 V

True/False Type

1. False **2.** True

Subjective Type

1. clockwise **2.** 1 mV
3. Yes, in the direction opposite to A.
4. (a) $\frac{1}{2}\frac{Br^2\omega}{R}$ (b) anticlockwise (c) see the solution
5. (a) $v = \frac{(R+2\lambda x)i}{Bd}$ (b) $F = \frac{2\lambda i^2 m}{B^2 d^2}(R+2\lambda x)^2 + idB$
6. (a) –5 V, 24.5 W
(b) (i) 0.6 A (ii) 1.386×10^{-3} s, 4.5×10^{-4} J
7. $\frac{7}{22}A(E \text{ to } A)$, $\frac{6}{22}A(B \text{ to } E)$, $\frac{1}{22}A(F \text{ to } E)$
8. $v = 1$ m/s, $R_1 = 0.47\ \Omega$, $R_2 = 0.3\ \Omega$
9. (a) $e = \frac{B\omega r^2}{2}$ (b) (i) $i = \frac{B\omega r^2}{2r}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$
(ii) $\tau_{net} = \frac{B^2\omega r^4}{4R} + \frac{mgr}{2}\cos\omega t$

10. 3.465 s

11. (a) $v = \frac{mgR}{B^2L^2}$ (b) $a = \frac{g}{2}$

12. $F = \frac{9}{4}\frac{\mu_0^2 M^2 a^4 v}{x^8 R}$

13. (a) 10^4 A/s (b) zero (c) 2.0 A (d) 1.732×10^{-4} C

14. (a) $i = \frac{B_0 a v}{R}$, anticlockwise (b) $\vec{F} = -\frac{B_0^2 a^2 v}{R}\hat{j}$

(c) $v = \frac{g}{K}(1 - e^{-kT})$ where $K = \frac{B_0^2 a^2}{mR}$, $v_t = \frac{g}{K} = \frac{gmR}{B_0^2 a^2}$

15. 35.6°C

16. $12e^{-5t}$ V, $6e^{-10t}A$ (clockwise)

17. (a) $\frac{d\phi}{dt} = iR + L\frac{di}{dt}$ (b) $\frac{1}{R}\left[\frac{\mu_0 I_0 l}{2\pi}\ln(2) - Li_1\right]$

(c) $\frac{T}{\ln(4)}$

18. $i_{max} = \frac{\mu_0 a C I_0 \omega^2 \ln(2)}{\pi}$

19. 20 A, $\frac{\pi}{4}$

20. $R = \frac{4}{3}(r_1 - r_2)$

21. 2 A, $\frac{\pi}{4}$

22. $i = \left(\frac{\mu_0 n i_0 \omega a^2 L d}{2\rho r}\right)\cos\omega t$

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. b. Effective resistance is 4 Ω.

$$I = \frac{E}{R} = \frac{B\ell v}{R} \quad \text{or} \quad v = \frac{IR}{B\ell}$$

or $$v = \frac{1\times 10^{-3}\times 4}{2\times 10\times 10^{-2}}\ \text{m s}^{-1}$$

$$= 0.02\ \text{m s}^{-1} = 2\ \text{cm s}^{-1}$$

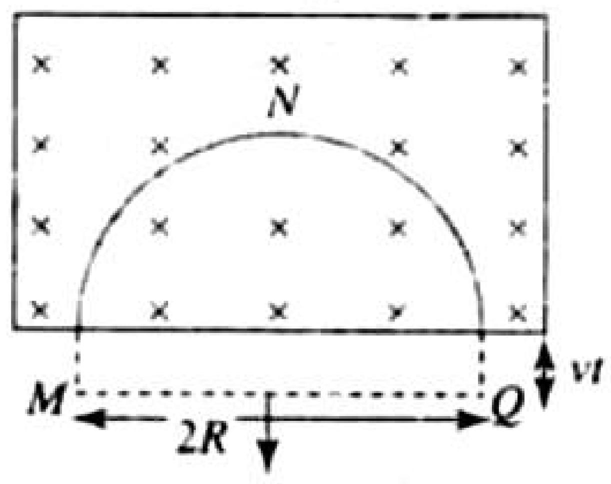
2. d. Net change in magnetic flux passing through the coil is zero.

∴ Current (or emf) induced in the loop is zero.

3. b. $$\text{Charge flown} = \frac{\text{Change in flux}}{\text{Resistance}} = \frac{BA-0}{R} = \frac{BA}{R}$$

4. (d) Rate of decrease of area of the semicircular ring

$$-\frac{dA}{dt} = (2R)V$$



According to Faraday's law of induction induced emf

$$e = -\frac{d\varphi}{dt} = -B\frac{dA}{dt} = -B(2RV)$$

The induced current in the ring must generate magnetic field in the upward direction. Thus Q is at higher potential.

5. b. A motional emf, $e = B\ell v$, is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod. Due to this potential difference, there is an electric field in the rod.

6. b. Magnetic field produced by a current I in a large square loop at its centre,

$$B \propto \frac{I}{L} \Rightarrow B = K\frac{I}{L}$$

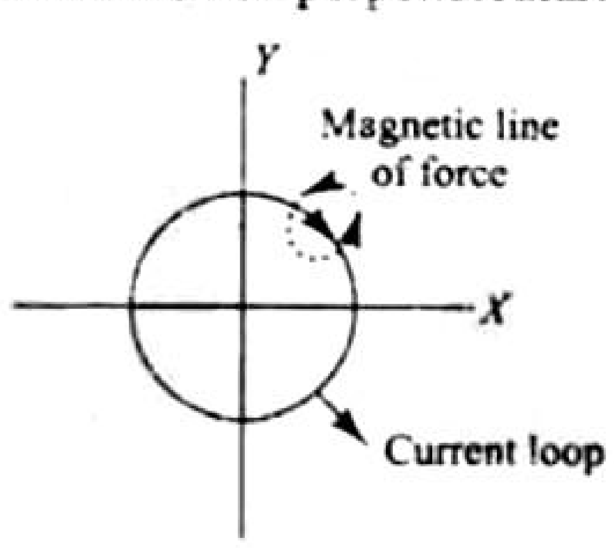
∴ Magnetic flux linked with the smaller loop,

$$\phi = BS \Rightarrow \phi = \left(K\frac{I}{L}\right)(l^2)$$

Therefore, the mutual inductance

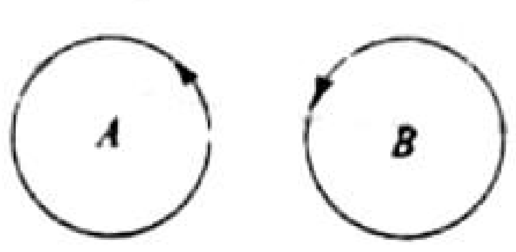
$$M = \frac{\phi}{I} = K\frac{\ell^2}{L} \quad \text{or} \quad M \propto \frac{\ell^2}{L}$$

7. d. The magnetic lines of force created due to current will be in such a way that on x–y plane these lines will be perpendicular. Further, these lines will be in circular loops. The number of lines moving downward in x–y plane will be the same in number to that coming upward of the x–y plane. Therefore, the net flux will be zero. One such magnetic line is shown in the figure.



∴ (d) is the correct option.

8. c. When the current in loop A increases, the magnetic lines of force in loop B also increase as loop A is near loop B.



This induces an emf in B is such a direction that current flows in opposite direction in B (as compared to A in the nearer parts). Since currents are in opposite directions, loop B is repelled by loop A.

9. d. We have

$$I = I_0(1 - e^{-t/\tau})$$

But $I_0 = \frac{V}{R}$ and $\tau = \frac{L}{R}$

$$\therefore \quad I = \frac{V}{R}(1 - e^{-Rt/L}) = \frac{12}{6}\left[1 - e^{-6t/8.4\times10^{-3}}\right]$$

$= 1$ (given)

$\therefore \quad t = 0.97 \times 10^{-3}$ s $= 1$ ms

10. b. $E(2\pi r) = \pi a^2 \frac{dB}{dt}$ for $r \geq a$

$$\Rightarrow \quad E = \frac{a^2}{2r}\frac{dB}{dt} \quad \Rightarrow \quad E \propto \frac{1}{r}$$

11. d. The magnetic field at the centre of the coil is $B(t) = \mu_0 n I_1$. As the current increases, B will also increase with time till it reaches a maximum value (when the current becomes steady). The induced emf in the ring

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(\vec{B}\cdot\vec{A})$$

$$= A\frac{d}{dt}(\mu_0 n I_1)$$

$\therefore$ The induced current in the ring

$$I_2(t) = \frac{|e|}{R} = \frac{\mu_0 nA}{R}\frac{dI_1}{dt}$$

$$I_2 B \propto I_1 \frac{dI_1}{dt}$$

$$\Rightarrow \quad I_2 B = KI_0\left[1 - e^{-t/\tau}\right]\left(\frac{I_0}{\tau}e^{-t/\tau}\right)$$

$$= \frac{KI_0^2}{\tau}\left[e^{-t/\tau}\right]\left[1 - e^{-t/\tau}\right]$$

at $t = 0$ and $t = \infty$, $I_2 B = 0$

12. d. Both AD and BC are straight conductors moving in a uniform magnetic field and emf will be induced in both. This will cause electric fields in both, but no net current flows in the circuit.

13. a. When current flows in any of the coils, the flux linked with the other coil will be maximum in the first case. Therefore, mutual inductance will be maximum in case (a).

14. d. When switch S is closed magnetic field lines passing through Q increases in the direction from right to left. So, according to Lenz's law induced current in Q, *i.e.*, I_{Q_1} will flow in such a direction so that the magnetic field lines due to I_{Q_2} passes from left to right through Q. This is possible when I_{Q_1} flows in anticlockwise direction as seen by E. Opposite is the case when switch S is opened, *i.e.*, I_{Q_2} will be clockwise as seen by.

15. b. $P_1 = \frac{e_1^2}{R_1}$, $e_2 = 4e_1$

$$R_1 = \frac{\rho\ell}{A},\ R_2 = \frac{\rho 4\ell}{A/4} = \frac{16\rho\ell}{A} = 16R_1$$

$$P_2 = \frac{e_2^2}{R_2} = \frac{(4e_1)^2}{16R_1} = \frac{e_1^2}{R_1} = P_1$$

16. a. Since current leads emf (as seen from the graph), therefore this is an R–C circuit.

$$\tan\phi = \frac{X_C - X_L}{R} \quad \text{Here, } \phi = 45°$$

$\therefore \quad X_C = R \qquad [X_L = 0$ as there is no inductor$]$

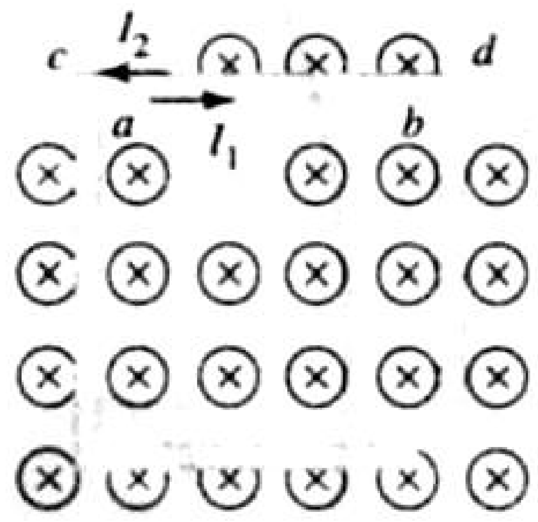
$$\frac{1}{\omega C} = R$$

$$\text{L.H.S.} = \frac{1}{100 \times 10 \times 10^{-6}} = R = 10^3\ \Omega$$

17. c. Initially ϕ_B increases as the magnet approaches the solenoid. Therefore, ε is negative and increases in magnitude when the magnet moves inside the solenoid. Increase in ϕ_B slows down and finally ϕ_B starts decreasing. Therefore e.m.f. becomes positive and starts increasing. Only graph (c) shows these characteristics. Also polarity of emf will change when magnet starts coming out.

18. a. For a current to induce in the cylindrical conducting rod, the cylindrical rod should cut magnetic lines of force which will happen only when the cylindrical conducting rod is moving. Since conducting rod is at rest, no current will be induced. Also, a changing magnetic field will create an electric field which can apply force on the free electrons of the conducting rod and a current will get induced. But since the magnetic field is constant, no current will be induced.

19. d. In this loop, the same current will flow throughout the circuit. B is increased, as current will flow in the inner circuit from a to b and in the outer circuit, I_2 is the same as I_1 and in the other direction. Current in the loop opposes the field B.



20. b. Impedance of the circuit,

$$Z = \sqrt{(X_C)^2 + (R)^2} = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$$

As ω increases, z decreases.

Current in the circuit, $I = \frac{V_0}{Z}$

When ω is increased, the impedance of the circuit decreases and the current through the bulb increases. Therefore the bulb glows brighter.

21. c. True for induced electric field and magnetic field.

Multiple Correct Answer Type

1. **d.** Since the rate of change of magnetic flux is zero, hence there will be no net induced emf and hence no current flowing in the loop.
2. **a., c., d.**

$$V_1 = L_1\frac{dI_1}{dt} \text{ and } V_2 = L_2\frac{dI_2}{dt}$$

But $\frac{dI_1}{dt} = \frac{dI_2}{dt}$ (given)

$$\therefore \quad \frac{V_1}{V_2} = \frac{L_1}{L_2} = \frac{8}{2} = \frac{4}{1}$$

Again, same power is given to the two coils.

$$\therefore \quad V_1 I_1 = V_2 I_2 \quad \text{or} \quad \frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{1}{4}$$

Again, energy $= \frac{1}{2} LI^2$

$$\therefore \quad \frac{W_2}{W_1} = \frac{\frac{1}{2} L_2 I_2^2}{\frac{1}{2} L_1 I_1^2} = \left(\frac{L_2}{L_1}\right)\left(\frac{I_2}{I_1}\right)^2 = \frac{2}{8}(4)^2 = \frac{4}{1}$$

3. a., b., c., d.

a. $L = \frac{\phi}{i}$ or henry $= \frac{\text{weber}}{\text{ampere}}$

b. $e = -L\left(\frac{di}{dt}\right)$

$$\therefore \quad L = -\frac{e}{(di/dt)}$$

or $\quad \text{henry} = \frac{\text{volt-second}}{\text{ampere}}$

c. $U = \frac{1}{2} Li^2$

$$\therefore \quad L = \frac{2U}{i^2}$$

or $\quad \text{henry} = \frac{\text{joule}}{(\text{ampere})^2}$

d. $\tau = \frac{L}{R}$

$\therefore \quad L = R\tau$ or henry = ohm-second

4. b., d. Electrostatic and gravitational field do not make closed loops.

5. b., d. As $\frac{d\phi}{dt}$ = emf is the same, the current induced in the ring will depend upon the resistance of the ring. Larger the resistivity, smaller the current.

6. b., c. Case (A): $Z_A = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Case (B): $Z_B = \sqrt{R^2 + \left(\frac{1}{\omega KC}\right)^2}$

So $\quad Z_B < Z_A$

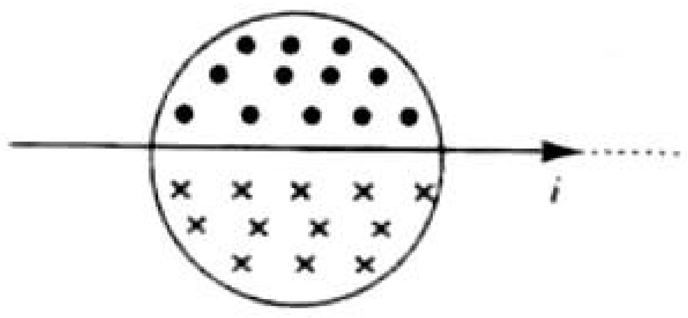
$$I_R^A = \frac{V}{Z_A} \quad \text{and} \quad I_R^B = \frac{V}{Z_B} \quad \text{clearly} \quad I_R^A < I_R^B$$

Since current in case (B) is greater, so p.d. across R will increase in case (B) and thus across capacitor will decrease.

Hence $V_C^A > V_C^B \qquad (\because \; V_R^2 + V_C^2 = V_0^2)$

7. a., c. $(\phi)_{loop} = 0$ for all cases

So induced emf = 0

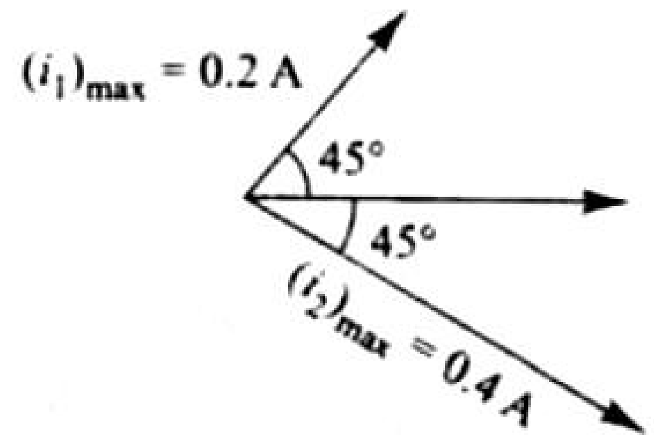


8. a., c. $C = 100\ \mu\text{F}$, $\frac{1}{\omega C} = \frac{1}{(100)(100 \times 10^{-6})}$

$$X_L = 100\ \Omega,\ X_L = \omega L = (100)(0.5) = 50\ \Omega$$

$$Z_1 = \sqrt{X_L^2 + 100^2} = 100\sqrt{2}\ \Omega$$

$$Z_2 = \sqrt{X_L^2 + 50^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}$$



$$\varepsilon = 20\sqrt{2}\sin \omega t$$

$$i_1 = \frac{1}{5}\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$i_2 = \frac{20\sqrt{2}}{50\sqrt{2}}\sin\left(\omega t - \frac{\pi}{4}\right) = \frac{2}{5}\sin\left(\omega t - \frac{\pi}{4}\right)$$

$$i = \sqrt{(0.2)^2 + (0.4)^2} = (0.2)\sqrt{1+4} = \frac{1}{5}\sqrt{5} = \frac{1}{\sqrt{5}}$$

$$(I)_{rms} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \approx 0.3\ \text{A}$$

$$(V_{100\Omega}) = (I_1)_{rms} \times 100 = \left(\frac{0.2}{\sqrt{2}}\right) \times 100 = 10\sqrt{2}\ \text{V}$$

$$(V_{50\Omega})_{rms} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{V}$$

9. c., d. If q represents the charge on capacitor's upper plate:

$$I(t) = I_0 \cos(\omega t) = \frac{dq}{dt} \quad \Rightarrow \quad q(t) = \frac{I_0}{\omega}\sin(\omega t)$$

Max charge $= \frac{1\ \text{A}}{500\ \text{rad s}^{-1}} = 2 \times 10^{-3}\text{C}$

Charge on upper plate at $t = \frac{7\pi}{6\omega} = \frac{I_0}{\omega}\sin\left(\frac{7\pi}{6}\right) = -\frac{I_0}{2\omega}$

When capacitor is fully charged, charge on upper plate = 50 V × 20 μF = 1 × 10^{-3}C

$$\therefore \quad Q = 1 \times 10^{-3}\text{C} - \left(-\frac{I_0}{2\omega}\right) = 2 \times 10^{-3}\text{C}$$

Voltage across capacitor when A and D are connected

$$= \frac{1 \times 10^{-3}\text{C}}{20 \times 10^{-6}\mu\text{F}} = 50\ \text{V}$$

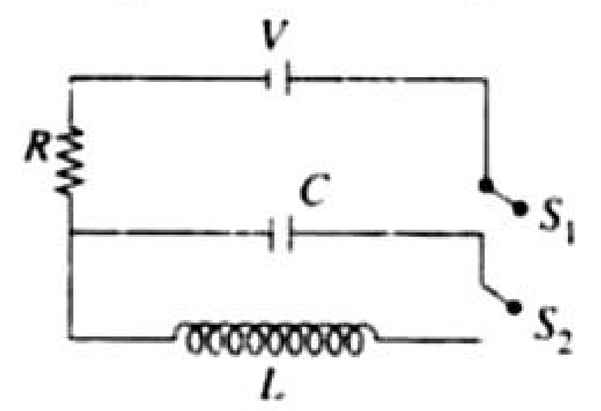
Total voltage across resistor = 100 V

$$\Rightarrow \quad \text{current} = \frac{100\ \text{V}}{10\ \Omega} = 10\ \text{A}$$

Current is anticlockwise.

Linked Comprehension Type

1 (b) At the start, the capacitor is uncharged. The charging occurs.



At time t,

$$q = q_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

$t = 2\tau$ (given)

or $\quad q = (CV)\left(1 - e^{-\frac{2\tau}{\tau}}\right)$

or $\quad q = (CV)(1 - e^{-2})$

$\therefore$ After time interval

$(2\tau), q = (CV)(1 - e^{-2})$

2. d. From conservation of energy,

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV^2 \Rightarrow I_{max} = V\sqrt{\frac{C}{L}}$$

3. c. Comparing the LC oscillation with normal SHM, we get

$$\frac{d^2Q}{dt^2} = -\omega^2 Q$$

Here, $\omega^2 = \frac{1}{LC}$

$$Q = -LC\frac{d^2Q}{dt^2}$$

4. a. The advantage of Maglev train is that there is no friction between the train and the track, thereby reducing power consumption.

The levitation of the train is due to magnetic repulsion.

5. d. The disadvantage of maglev trains is that initial cost is high.

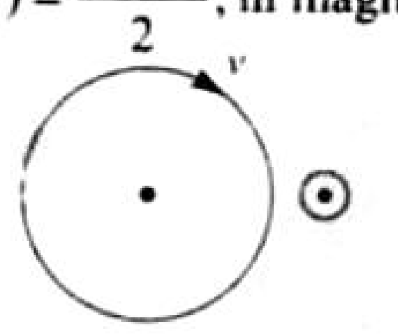
6. b. The magnetic force will pull the vehicle.

7. b. $E(2\pi R) = \pi R^2 \frac{dB}{dt}$

$$E = \frac{RB}{2}$$

8. b. $\Delta L = \int \tau dt$

$$= Q\left(\frac{R}{2}B\right)R(1) = \frac{QR^2B}{2}, \text{ in magnitude}$$



$$\Delta\mu = \gamma\Delta L$$

$$= -\gamma\frac{BQR^2}{2} \text{ (taking into account the direction)}$$

9. b. We can define power, $P = IV$

$$\therefore \quad I = \frac{P}{V} = \frac{600 \text{ kW}}{4000 \text{ V}} = \frac{600 \times 10^3 \text{ W}}{4000 \text{ V}} = 150 \text{ A}$$

Resistance of the cable,

$$R = 0.4\,\Omega\,\text{km}^{-1} \times 20 \text{ km} = 8\,\Omega$$

Line power loss $= I^2R = (150 \text{ A})^2(8\Omega)$

$= 180 \times 10^3\,\Omega = 180 \text{ k}\Omega$

$$\% \text{ loss} = \frac{180 \text{ kW}}{600 \text{ kW}} \times 100 = 30\%$$

10. a. For a transformer, $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

For a step-up transformer,

Here, $\frac{N_p}{N_s} = \frac{1}{10}, V_p = 4000$ V

$$\therefore \quad V_s = V_p\frac{N_s}{N_p} = (4000V) = 40{,}000 \text{ V}$$

For a step-down transformer,

Here, $\frac{N_p}{N_s} = ?, V_p = 40{,}000 \text{ V}, V_s = 200$ V

$$\therefore \quad \frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40{,}000 \text{ V}}{200 \text{ V}} = \frac{200}{1}$$

$$N_p : N_s = 200 : 1$$

Matching Column Type

1. i. →a.; ii. → a., b., d.; iii. → b., d.; iv. → b., c.

i. A uniformly charged ring produces time independent electrostatic field.
Hence i.→a.

ii. A uniformly charge dielectric ring which is rotating with same angular velocity is equivalent to a circular current carrying loop and this can produce electric field, magnetic field and magnetic moment.
Hence ii. → a., b.,

iii. Constant current in a ring can produce magnetic field and has magnetic moment.
Hence iii. → b.

iv. If time dependent current flow in a solenoid, it can produce time dependent magnetic field. The time dependent magnetic field can produce induced electric field.
Hence iv. → b., c.

2. i. → b.; ii. → c., d.; iii. → d.; iv. → a., b., c.

i. If a charge capacitor is connected to the ends of a resistance, a time dependent current starts flowing in the wire. It produces heating in the resistance.
Hence i. → b.

ii. If the given wire moves perpendicular to its length in a magnetic field perpendicular to the plane of motion, the motional emf develop in the wire. This emf develops a constant potential difference between the ends of wire. Because this emf an electric field develops in the wire, hence charge of constant magnitude develops at the end of wire.
Hence ii → c., d.;

iii. If the wire is placed in a constant electric field in such a way that its length is parallel to the direction of electric field. The free electrons in the wire moves in the direction opposite to the field. Hence charges of constant magnitude appears at the ends of the wire.
Hence iii. → d.;

iv. If a battery of constant emf is connected to the ends of the wire. A potential difference develops across the wire, can simply observe a constant current will starts flowing in the wire. If a current flows in a resistance, there will be heat develops in the wire.
Hence iv. → a., b., c.

3. **(i) – (c); (d); (e); (ii) – (b), (c), (d), (e); (iii) – (a), (b); (iv) – (b), (c), (d), (e).**

a. In this case, steady state current is zero ($I = 0$) because of the presence of capacitor. Entire potential will be across capacitor in steady state. Hence $V_1 = 0$ and $V_2 = V$.

b. In steady state $V_1 = 0$, so $V_2 = 0$. Also V_2 is proportional to I.

c. $X_L = \omega L = 2\pi f L = 2\pi \times 50 \times 6 \times 10^{-3} = 1.885\ \Omega$

$R = 2\ \Omega$

Since $R > X_L$, so $V_2 > V_1$

Here $I \neq 0$

V_1 and V_2 both are proportional to I.

d. $X_L = 1.885\ \Omega$.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 3 \times 10^{-6}} = 1061\ \Omega$$

Hence $X_C > X_L$, so $V_2 > V_1$

e. $X_C = 1.061\ \text{k}\Omega$, $R = 1\ \text{k}\Omega$

$X_C > R$, so $V_2 > V_1$

Integer Answer Type

1. (6) Flux through circular ring

$$\phi = \left(\frac{\mu_0 I}{L}\right)\pi r^2 \Rightarrow \phi = \frac{\mu_0}{L}\pi r^2 I_0 \cos 300t$$

Current in the ring $i = \frac{1}{R}\frac{d\phi}{dt}$

$$i = \frac{\mu_0 \pi r^2 I_0}{RL} \cdot \sin 300t \times 300$$

$$= \mu_0 I_0 \sin 300t \left[\frac{\pi r^2 \cdot 300}{RL}\right]$$

$$M = I \cdot \pi r^2 = \mu_0 I_0 \sin 300t \left[\frac{\pi^2 r^4 \cdot 300}{RL}\right]$$

Taking $\pi^2 = 10$, we get

$M = 6\mu_0 I_0 \sin 300t$

thus $N = 6$

2. (4) $\omega = 500\ \text{rad s}^{-1}$

$$Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = R^2(1.25) \Rightarrow \left(\frac{1}{\omega C}\right)^2 = \frac{R^2}{4}$$

$$\Rightarrow \quad CR = \frac{2}{\omega} = \frac{2}{500}\ \text{s} = \frac{2}{500} \times 10^3\ \text{ms} = 4\ \text{ms}$$

3. (7) $B = \dfrac{\mu_0 i R^2}{2(R^2 \times X^2)^{3/2}}$

$$B = \frac{\mu_0 i R^2}{2(R^2 \times 3R^2)^{3/2}} = \frac{\mu_0 i R^2}{2(4R^2)^{3/2}} = \frac{\mu_0 i R^2}{2 \cdot 2^3 R} = \frac{\mu_0 i}{16R}$$

$$\phi = 2\frac{\mu_0 i}{16R} a^2 \frac{1}{\sqrt{2}} = \frac{\mu_0 i a^2}{8\sqrt{2}R}$$

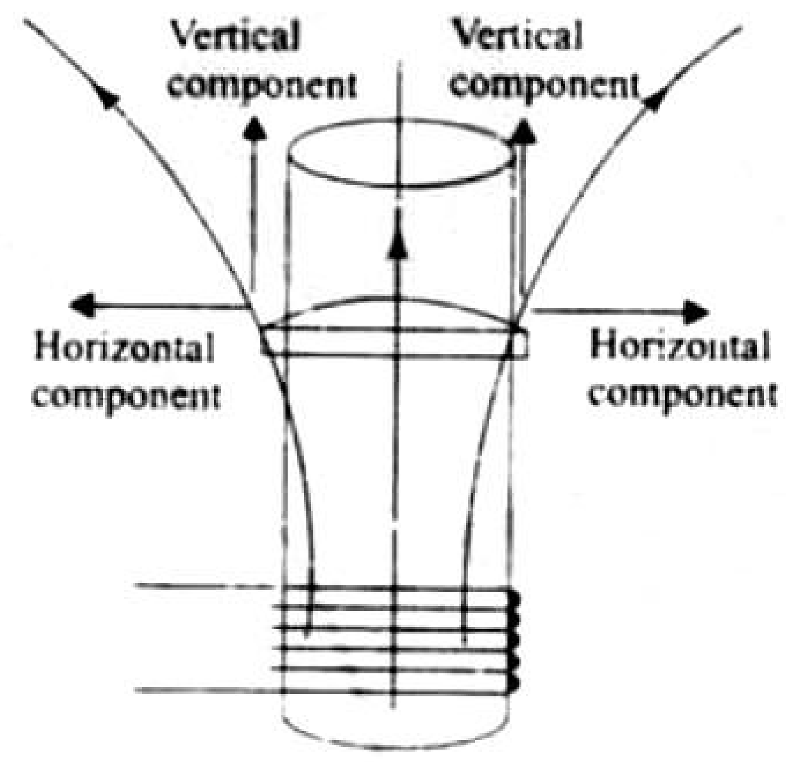
$$M = \frac{\phi}{i}$$

$$M = \frac{\mu_0 a^2}{2^{7/2} R} = \frac{\mu_0 a^2}{2^{p/2} R}$$

$p = 7$

Assertion-Reasoning Type

1. a. The induced current in the ring will interact with horizontal component of magnetic field and both will repel each other. This repulsion will balance the weight of the ring.



Fill in the Blanks Type

1. The coil is broken into two identical coils.

$$L_{eq} = \frac{L/2 \times L/2}{L/2 + L/2} = \frac{L}{4} = 0.45 \times 10^{-4}\ \text{H}.$$

$$R_{eq} = \frac{R/2 \times R/2}{\frac{R}{2} + \frac{R}{2}} = \frac{R}{4} = 1.5\ \Omega$$

$$\text{Time constant} = \frac{L_{eq}}{R_{eq}} = \frac{0.45 \times 10^{-4}}{1.5} = 0.3 \times 10^{-4}\ \text{s}.$$

Steady current.

$$I = \frac{E}{R_{eq}} = \frac{12}{1.5} = 8\ \text{A}$$

2. We know that the velocity of light in vacuum $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and the velocity of light in a medium $v = \frac{1}{\sqrt{\mu\varepsilon}}$. Also, the refractive index

$$n = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}} = \frac{c}{v}$$

$$= \frac{1/\sqrt{\mu_0 \varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0 \varepsilon_0}}$$

3. Left to right, apply Lenz's law

4. $V_A - IR + E - L\dfrac{dI}{dt} = V_B$

$\Rightarrow \quad V_A - 5 \times 1 + 15 - 5 \times 10^{-3}(-10^{-3}) = V_B$

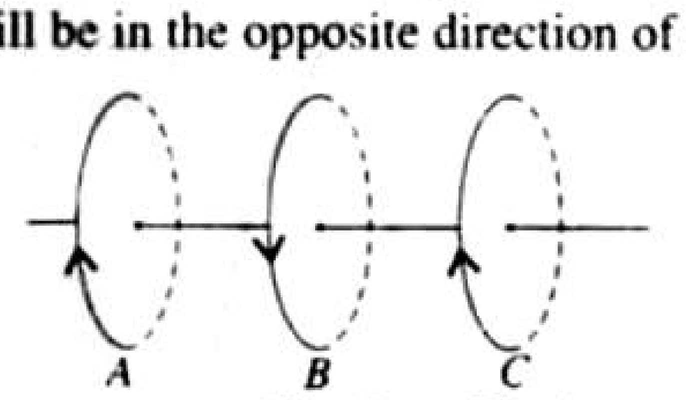
$\Rightarrow \quad V_B - V_A = 15\ \text{V}$

True/False Type

1. For induced emf to develop in a coil, the magnetic flux through it must change with time. But in this case the number of magnetic lines of force through the coil is not changing. Therefore the statement is false.
2. When conducting rod AB moves parallel to x-axis in a uniform magnetic field pointing in the positive z direction, then according to Fleming's left hand rule, the electrons will experience a force towards B. Hence, the end A will become positive. Therefore, the statement is true.

Subjective Type

1. Magnetic field due to straight wire passing through the wire loop will be perpendicular to paper outsides. With increase in current in straight wire, outwards magnetic field through the loop will increase. Therefore, from Lenz's law, inward magnetic field will be produced by the induced current. Hence, induced current is clockwise.
2. This is based on motional emf. $e = vBl$ $50 \times 0.2 \times 10^{-4} \times 1 = 10^{-3}$ volt = 1 millivolt
3. When the coil A move towards B, the number of magnetic lines of force passing through B changes. Therefore an induced emf and hence induced current in produce in B. The direction of current in B will be such as to oppose the field change in B and therefore will be in the opposite direction of A.

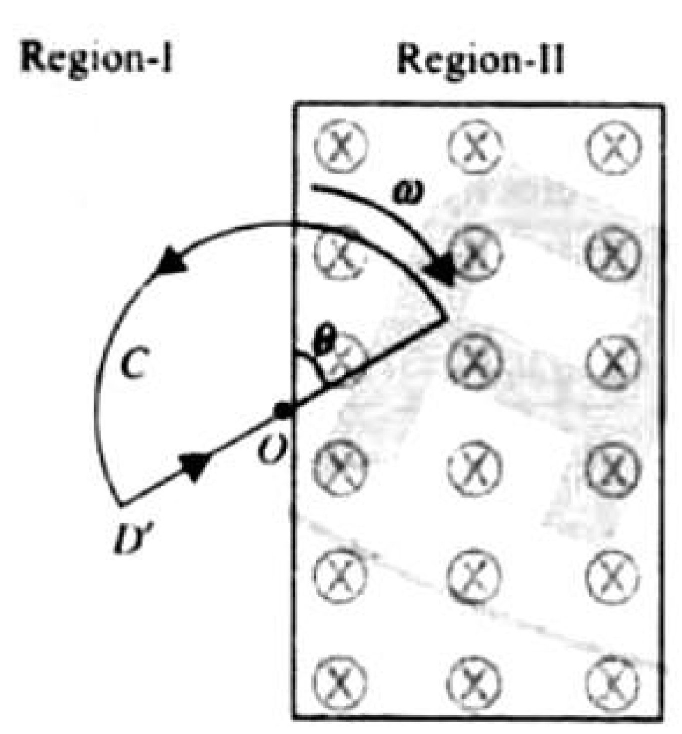


4. a. When the loop is rotated about an axis passing through center O and perpendicular to the plane of the paper, the angle between magnetic field vector $\vec{B}$ and area $\vec{A}$ is always 0°. When the loop is in region I, the magnetic flux linked with loop = $BA \cos 0 = 0$ the (since $B = 0$ in region I).

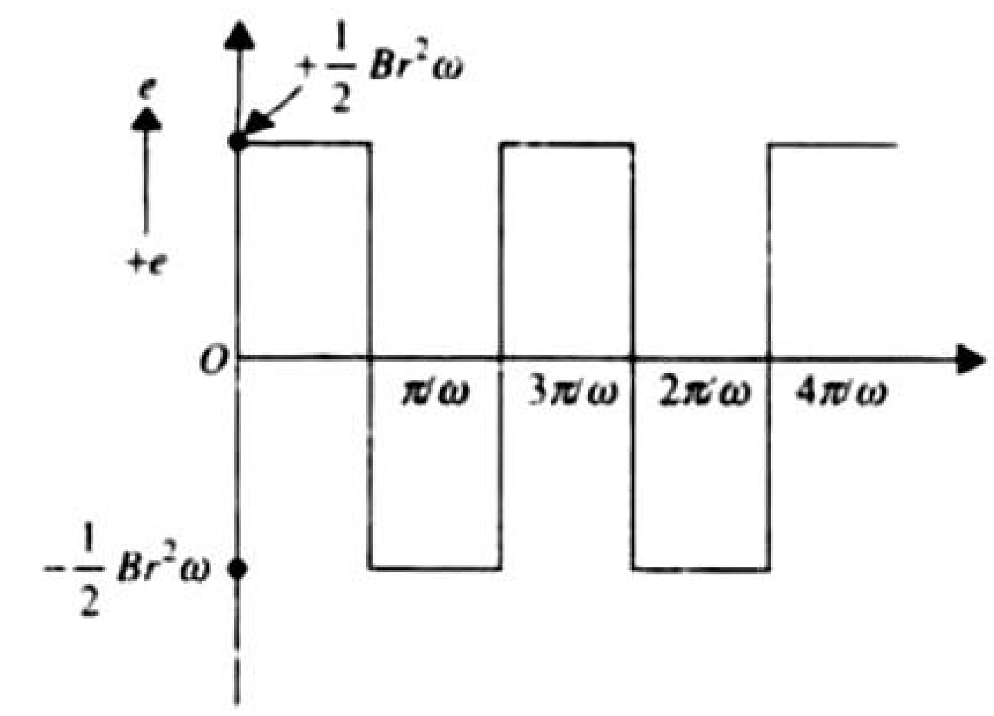
When the loop enters the magnetic field in region II, the magnetic flux linked with it is given by $\phi = BA$ where $A = \frac{1}{2}r^2\theta$.

Therefore, emf induced

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt} = \frac{-Br^2}{2}\frac{d\theta}{dt} = -\frac{Br^2}{2}\omega$$



(a)



(b)

As resistance of the loop is R, the current induced is given by

$$i = \frac{e}{R} = \frac{1}{2}\frac{Br^2\omega}{R}$$

This is the required expression for current induced in the loop.

b. According to Lenz's law, the direction of current induced is to oppose the change in magnetic flux. So, when entering into region II the field produced by the current induced must be upward. For this, the current in the loop must be anticlockwise as shown in Figure (a).

c. When the loop enters the magnetic field, the magnetic flux linked with it increases and the emf $e = \frac{1}{2}Br^2\omega$ is induced in one direction. When the loop comes out of the field, the flux decreases and emf is induced in opposite sense. The graph for representing the emf induced versus time for two periods ($T = 2\pi/\omega$) is shown in Figure (b). Here we have taken anticlockwise direction as positive.

5. a. If the rod has instantaneous velocity v at a distance x from R, the induced emf is Bvd.

Instantaneous resistance of current = $R + 2\lambda x$

$\therefore$ Induced current $i = \frac{Bvd}{R + 2\lambda x}$ = constant

$\therefore$ Velocity $v = \frac{(R + 2\lambda x)i}{Bd}$

Magnetic force on the rod = Bid.

This force will be opposite to F. Hence, net force acting on rod,

$$F - Bid = m\frac{dv}{dt}$$

or $$F - Bid = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{d}{dx}\left\{\frac{(R + 2\lambda x)i}{bd}\right\} = mv\frac{2\lambda i}{Bd}$$

$$\therefore \quad F = Bid + 2m\lambda\frac{(R + 2\lambda x)}{B^2d^2}i^2$$

b. Work done per second = Fv

Heat produced per second = $i^2(R + 2\lambda x)$

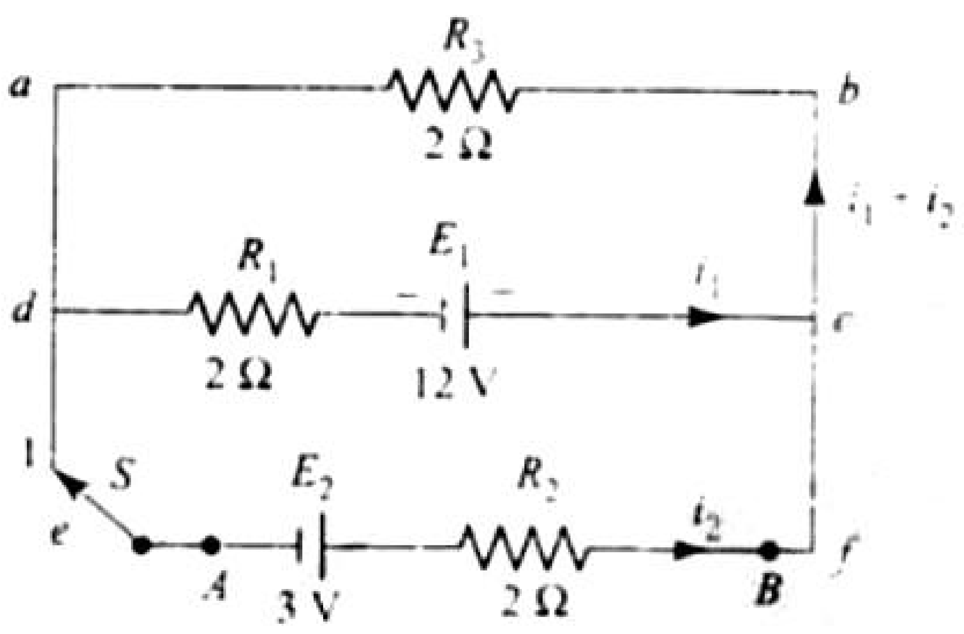
$$\text{Required ratio} = \frac{i^2(R + 2\lambda x)}{Fv} = \frac{i^2(R + 2\lambda x)Bd}{F(R + 2\lambda x)i} = \frac{iBd}{F}$$

$$= \frac{Bid}{Bid + \frac{2m\lambda(R + 2\lambda x)i^2}{B^2d^2}}$$

$$= \frac{1}{1 + \frac{2m\lambda(R + 2\lambda x)i}{B^3d^3}}$$

6. a. The capacitor offers infinite resistance to dc in the steady state, therefore, the current in capacitor branch is zero. The equivalent circuit, in steady state, when switch is in position (1) is given in the figure.

The distribution of current according to Kirchhoff's first law is shown in the figure. Applying Kirchhoff's second law to mesh *abcda*,



$$i_1 \times 2 + (i_1 + i_2) \times 2 = 12$$

or $\quad 4i_1 + 2i_2 = 12$ or $2i_1 + i_2 = 6 \qquad$ (i)

Applying Kirchhoff's second law to mesh *abfea*,

$$i_2 \times 2 + (i_1 + i_2)\, 2 = 3 \text{ or } 2i_1 + 4i_2 = 3 \qquad \text{(ii)}$$

Solving Eqs (i) and (ii), we get, $i_1 = 3.5$ A, $i_2 = -1$ A

∴ Potential difference between *A* and *B* is

$$V_B - V_A = V = E_2 - i_2R_2 = 3 - (-1) \times 2 = 5 \text{ V}$$

∴ $\quad V_A - V_B = -5$ V

b. When switch *S* is put in position 2, the equivalent circuit takes the form as shown in the figure.

Total resistance of the circuit,

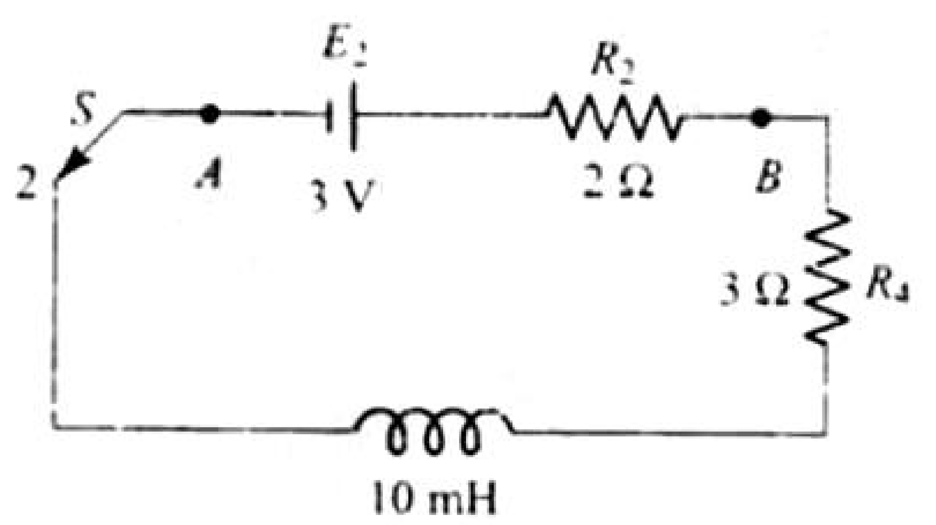
$$R = R_2 + R_4 = 2 + 3 = 5\ \Omega$$

$$L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H}$$

In steady state, there is no role of inductor *L*.

∴ Steady current

$$i_0 = \frac{E}{R} = \frac{3}{5} = 0.6 \text{ A}$$



The growth of current in *RL* circuit is given by

$$i = i_0\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{i.e.,} \quad \frac{i}{i_0} = \left(1 - e^{-\frac{R}{L}t}\right)$$

Given $i = \frac{i_0}{2}$, i.e., $\frac{i}{i_0} = \frac{1}{2}$ after time *t*.

$$\therefore \quad \frac{1}{2} = 1 - e^{-\frac{R}{L}t} \quad \text{or} \quad e^{-\frac{R}{L}t} = \frac{1}{2}$$

i.e., $\quad \frac{R}{L}t = \log_e 2$

i.e., $\quad t = \frac{L}{R}\log_e 2 = \frac{10 \times 10^{-3}}{5}\log_e 2 = 1.38 \times 10^{-3}$ s

The current at the instant, $i = i_0/2 = 0.3$ A

∴ Energy stored,

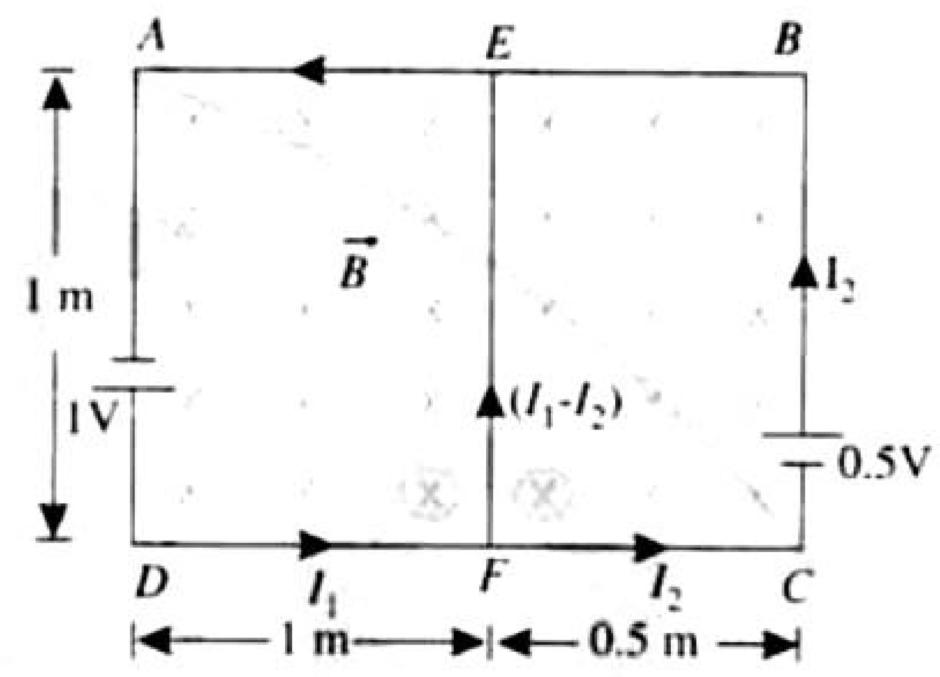
$$U = \frac{1}{2}Li^2 = \frac{1}{2} \times 10 \times 10^{-3} \times (0.3)^2 = 4.5 \times 10^{-4} \text{ J}$$

7. Given $\frac{dB}{dt} = +\frac{1T}{s}$

According to Faraday's law, induced e.m.f

$$E = \frac{d\phi}{dt} = \frac{d}{dt}(BA)$$

In the given problem, areas of two circuits remain constant.



$$\therefore \quad e = A\frac{dB}{dt}$$

For circuit *AEFD*, $e_1 = (1 \times 1)^2$ 1T/s = 1 volt

for circuit *EBCF* = $(0.5 \times 1 \text{ m}^2) \times$ 1T/s = 0.5 volt

According to Lenz's law, the current induced opposes the increase of current, hence current induced will tend to produce magnetic field in upward direction perpendicular to the plane of paper. Hence currents induced are shown in the figure.

As resistance of wire = 1 Ω/m

$$R_{BC} = R_{AE} = R_{EF} = R_{DF} = R_{AD} = 1 \times 1 = 1\ \Omega$$

$$R_{BE} = R_{FC} = 0.5 \text{ m} \times 10\ \Omega/\text{m} = 0.5\ \Omega$$

Applying Kirchhoff's second law of mesh *AEFDA* (induced e.m.f = 1 V)

$$I_1 \times 1 + (I_1 - I_2) \times 1 + I_1 \times 1 + I_1 \times 1 = 1 \text{ V}$$

or $\quad 4I_1 - I_2 = 1 \qquad$ (i)

Applying Kirchhoff's II law of mesh *BEFCB* (induced emf = 0.5 V)

$I_1 \times 0.5 + I_2 \times 1 + I_2 \times 0.5 - (I_1 - I_2) \times 1 = 0.5 - I_1 + 3I_2 = 0.5 \qquad$ (ii)

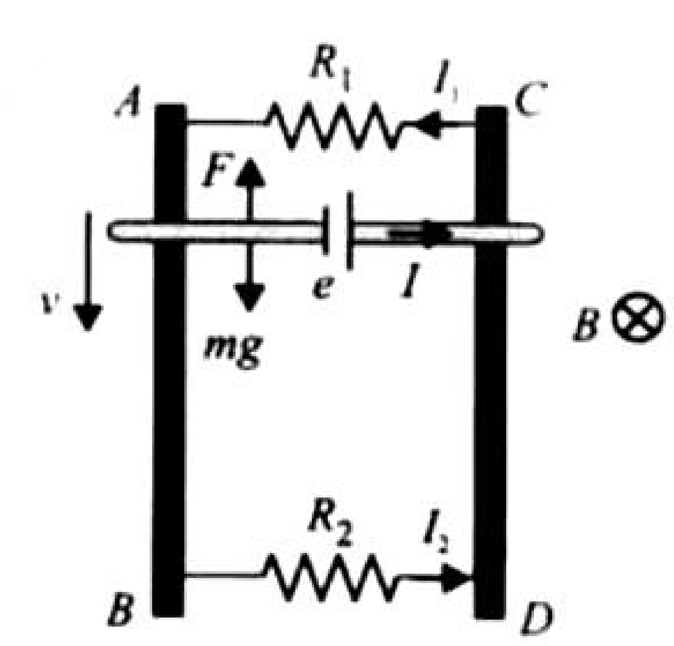
Solving (i) and (ii) $I_1 = \frac{7}{22}$ A, $I_2 = \frac{3}{11}$ A

Hence current in segment *AE*, $I_1 = \frac{7}{22}$ A, directed from *E*.

Current in segment $EF = I_1 - I_2$

$$= \frac{7}{22} - \frac{3}{11} = \frac{1}{22} \text{ A, directed from } F \text{ to } E.$$

8. Let magnetic field $B = 0.6$ T be inward as shown in the figure. When the rod moves down due to gravity, emf is induced in the rod as shown. Due to this, current *I* flows in the rod which results in force *F* on the rod in upward direction. Let at any instant of time, velocity of rod be *v*.



Then $e = Bvl$ and $I = \frac{e}{R_{eq}}$,

where $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

$$F = IlB = \frac{e}{R_{eq}} lB = \frac{B^2 l^2 v}{R_{eq}}$$

Initially, F is less than mg. But as the velocity increases, F also increases. At a certain time, F will be equal to mg. At that time net force on the rod will become zero and velocity of rod will become constant. This constant velocity is known as terminal velocity. Let it be v_t. So putting $F = mg$

$$\Rightarrow \quad \frac{B^2 l^2 v_t}{R_{eq}} = mg \qquad \text{(i)}$$

But at this time, power disspated in resistances is

$$P_1 + P_2 = 0.76 + 1.2 = 1.96 \text{ W}$$

So we have $\frac{e^2}{R_{eq}} = P_1 + P_2 \Rightarrow \frac{B^2 l^2 v_t^2}{R_{eq}} = 1.96$ (ii)

From Eqs. (i) and (ii) $v_t = \frac{1.96}{mg} = \frac{1.96}{0.2 \times 9.8} = 1 \text{ m s}^{-1}$

From Eqs. (i) $R_{eq} = \frac{B^2 l^2 v_t}{mg} = \frac{(0.6)^2 \times 1^2 \times 1}{0.2 \times 9.8} = \frac{9}{49} \Omega$

$$\frac{P_1}{P_2} = \frac{e^2/R_1}{e^2/R_2} \Rightarrow \frac{0.76}{1.20} = \frac{R_2}{R_1} \Rightarrow \frac{19}{30} = \frac{R_2}{R_1}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{49}{9} = \frac{1}{R_1} + \frac{30}{19R_1}$$

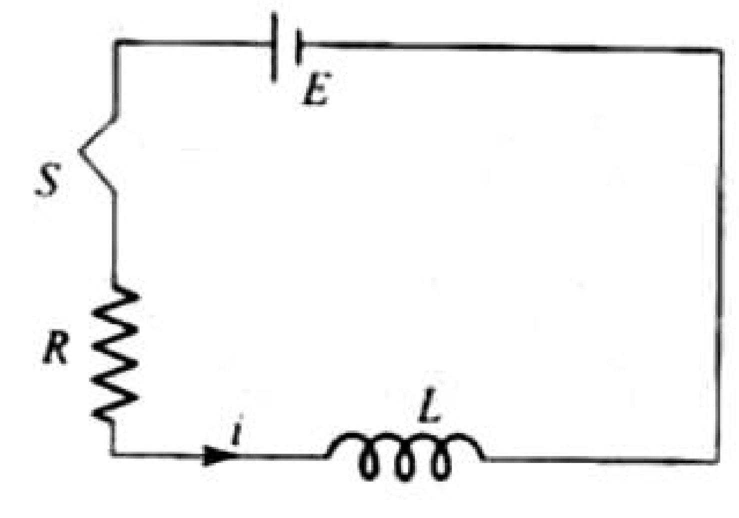
$$\Rightarrow \quad R_1 = \frac{9}{19} \Omega \text{ and } R_2 = \frac{19}{30} R_1 = \frac{19}{30} \times \frac{9}{19} = 0.3 \Omega$$

9. a. The emf induced across the whole rod is given by:

$$E = \frac{1}{2} B\omega r^2$$

$$= B\omega \left[\frac{x^2}{2}\right]_0^r = \frac{1}{2} B\omega r^2 \qquad \text{(i)}$$

b. i. When the rod rotates anticlockwise (as shown), the end O becomes positive and A negative. As resistance of ring and rod is negligible, therefore, the equivalent circuit is shown in the figure. When switch S is closed, let i be the current and di/dt the rate of change of current in the circuit, then from Kirchhoff's second law, the equation of emf is

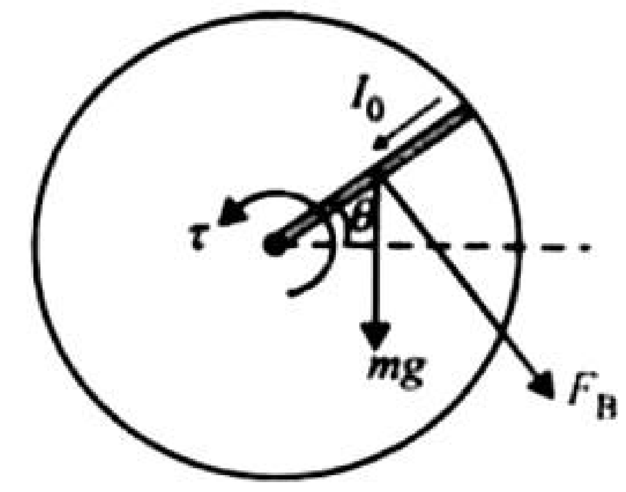


$$E = Ri + L \frac{di}{dt}$$

which upon integration gives $i = \frac{E}{R}(1 - e^{-Rt/L})$

Using Eq. (i), $i = \frac{Br^2\omega}{2R}(1 - e^{-Rt/L})$

ii. In steady state, current is $I_0 = E/R$



Magnetic force, $F_B = I_0 rB = \frac{ErB}{R}$, $\theta = \omega t$

Required torque

$$\tau = \tau_{\text{due to } F_B} + \tau_{\text{due to mg}}$$

$$\Rightarrow \quad \tau = F_B \frac{r}{2} + mg\frac{r}{2}\cos\theta = \frac{EBr^2}{2R} + \frac{mgr}{2}\cos\omega t$$

$$= \frac{B^2\omega r^4}{4R} + \frac{mgr\cos\omega t}{2}$$

10. $U = \frac{1}{2} Li^2$ i.e., $U \propto i^2$

U will reach 1/4[th] of its maximum value when current is reached half of its maximum value. In L-R circuit, equation of current growth is written as

$$i = i_0(1 - e^{-t/\tau_L})$$

Here i_0 = Maximum value of current

τ_L = Time constant = L/R

$$\tau_L = \frac{10H}{2\Omega} = 5 \text{ s}$$

Therefore, $i = \frac{i_0}{2} = i_0(1 - e^{-t/5})$

$$\frac{1}{2} = 1 - e^{-t/5}$$

$$e^{-t/5} = \frac{1}{2}$$

$$-t/5 = \ln\left(\frac{1}{2}\right)$$

$$t/5 = \ln(2) = 0.693$$

$$t = (5)(0.693)$$

$$t = 3.465 \text{ s}$$

11. a. If v is the instantaneous velocity of the rod at any time t, the induced emf will be $\varepsilon = BvL$.

∴ Induced current in rod

$$i = \frac{\varepsilon}{R} = \frac{BvL}{R} \qquad \text{(i)}$$

Due to this current, the rod in magnetic field B will experience a force

$$F = BiL \text{ opposite to motion.} \qquad \text{(ii)}$$

If v_T is terminal velocity of rod, and T the tension in the string, then free-body diagrams of rod and mass m are shown in the figure.

So equation of motion of mass m is $mg - T = ma$ (iii)

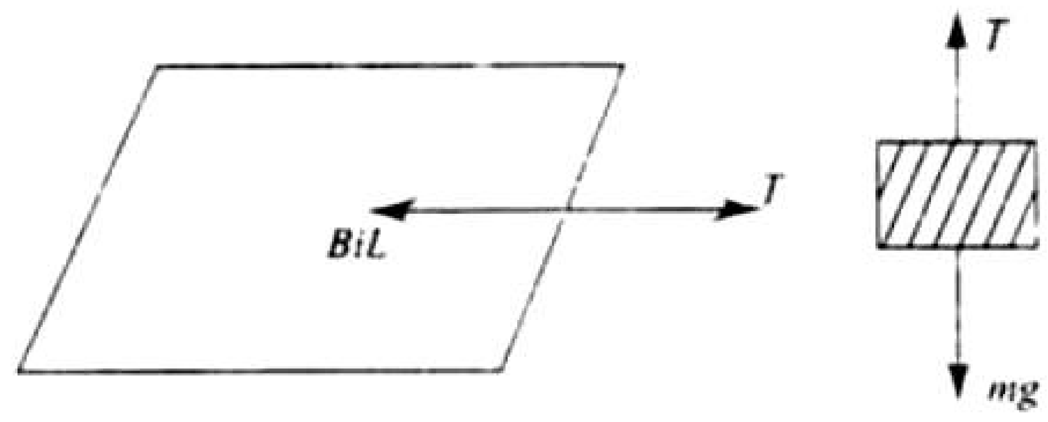
For terminal velocity, $a = 0$

$$mg - T = 0 \Rightarrow T = mg \qquad \text{(iv)}$$

Equation of motion of rod is

$T - BiL = 0 \times a = 0$ (as rod is massless)

$T = BiL$. (v)



Using Eq. (iv), we get, $mg = BiL$

$$mg = B\left(\frac{Bv_T L}{R}\right)L \Rightarrow v_T = \frac{mgR}{B^2L^2} \quad \text{(vi)}$$

b. When velocity of the rod is half the terminal velocity

$$v = \frac{v_T}{2} = \frac{1}{2}\frac{mgR}{B^2L^2} \quad \text{(vii)}$$

From Eq. (iii), acceleration

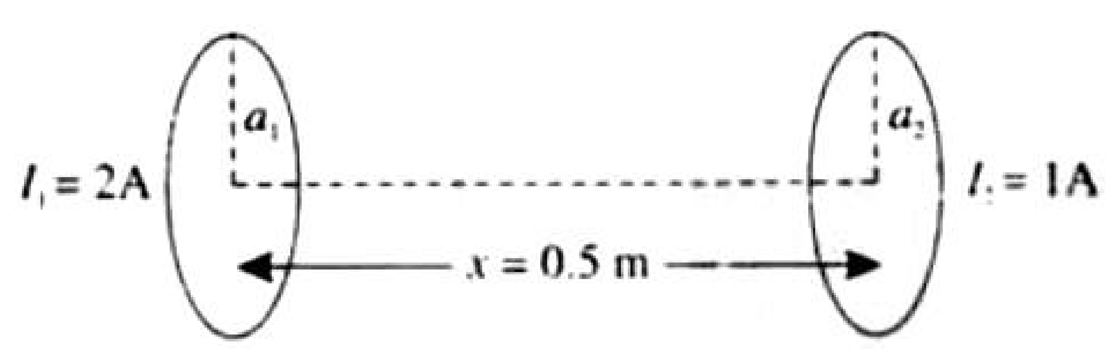
$$A = g - \frac{T}{m} = g - \frac{BiL}{m} \quad \text{[using v]}$$

$$= g - \frac{BL}{m}\frac{BL}{R}\frac{\frac{1}{2}mgR}{B^2L^2} \quad \text{[using (i) and (vii)]}$$

$$= g - \frac{g}{2} = \frac{g}{2}$$

12. The magnetic field produced by bar magnet at the location of the circular coil

$$B = \frac{\mu_0}{4\pi}\frac{2m}{x^3}$$



The magnetic flux linked with the coil of area A is

$$\phi = BA = \frac{\mu_0}{4\pi}\frac{2m}{x^3}A$$

The rate of change of magnetic flux linked with the coil

$$\frac{d\phi}{dt} = \frac{\mu_0}{4\pi}2MA\frac{d}{dx}(x^{-3})$$

$$= \frac{\mu_0}{4\pi}2MA(-3x^{-4})\frac{dx}{dt}$$

$$= -\frac{\mu_0}{4\pi}\frac{6MA}{x^4}v \quad \left(\text{since } v = \frac{dx}{dt}\right)$$

$$\therefore \quad \text{EMF induced, } \varepsilon = -\frac{d\phi}{dt} = \frac{\mu_0}{4\pi}\frac{6MA}{x^4}v$$

Current induced in the coil.

$$i = \frac{\varepsilon}{R} = \frac{\mu_0}{4\pi}\frac{6MAv}{x^4R}$$

Magnetic moment of coil, $M' = iA$

$$= \frac{\mu_0}{4\pi}\frac{6MA^2}{x^4R}v$$

The force between two magnetic dipoles at separation x is given by

$$F = \frac{\mu_0}{4\pi}\frac{6MM'}{x^4} = \frac{\mu_0}{4\pi}\frac{6M}{x^4}\left(\frac{\mu_0}{4\pi}\frac{6MA^2v}{x^4R}\right)$$

$$= \left(\frac{\mu_0}{4\pi}\right)^2\frac{36M^2A^2v}{x^8R}$$

As $A = \pi a^2$

$$\therefore \quad F = \left(\frac{\mu_0}{4\pi}\right)^2\frac{36M^2\pi^2a^4v}{x^8R} = \frac{9}{4}\frac{\mu_0^2M^2a^4v}{x^8R}$$

13. $L = 2.0$ mH $= 2.0 \times 10^{-3}$ H

$C = 5.0\ \mu$F $= 5.0 \times 10^{-6}$ F

$Q_{max} = 200\ \mu$C $= 200 \times 10^{-6}$ C

In an LC circuit, energy transfer continues from inductance to capacitance and vice versa.

a. By Kirchhoff's law in an LC circuit

$$\frac{Q}{C} + L\frac{dI}{dt} = 0 \Rightarrow \frac{dI}{dt} = -\frac{Q}{LC}$$

$$\therefore \left|\frac{dI}{dt}\right| = \frac{Q}{LC} = \frac{100 \times 10^{-6}}{2.0 \times 10^{-3} \times 5.0 \times 10^{-6}} = 10^4\ \text{A s}^{-1}$$

b. When $Q = 200\ \mu$C, the entire energy of circuit resides in capacitance. That is, no energy is stored in inductance.

$$\therefore \quad \frac{1}{2}LI^2 = 0 \Rightarrow I = 0$$

c. Maximum value of I is given by

$$\frac{1}{2}LI_{max}^2 = \frac{Q_{max}^2}{2C} \Rightarrow I_{max} = \frac{1}{\sqrt{LC}}Q_{max}$$

$$\text{or } I_{max} = \frac{1}{\sqrt{(2.0 \times 10^{-3}) \times (5.0 \times 10^{-6})}} \times 200 \times 10^{-6} = 2\ \text{A}$$

d. Given $I = \frac{I_{max}}{2} = \frac{2}{2} = 1$ A

Then again from the conservation of energy.

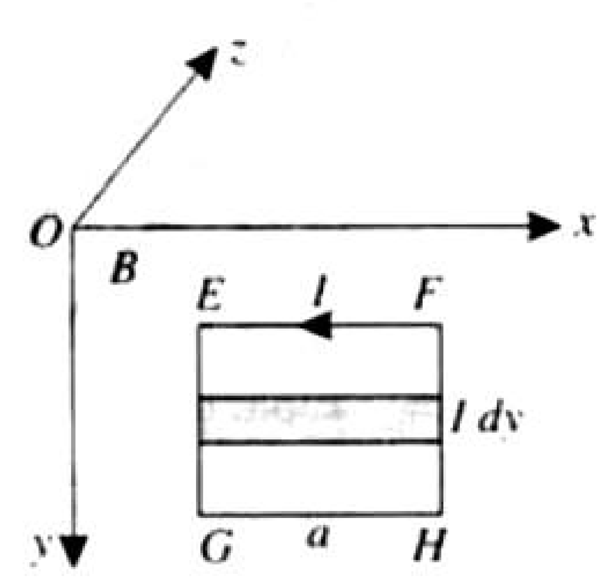
$$\frac{1}{2}LI^2 + \frac{Q^2}{2C} = \frac{1}{2}LI_{max}^2; \frac{Q^2}{2C} = \frac{1}{2}L\left(I_{max}^2 - I^2\right)$$

$$Q = \sqrt{LC\left(I_{max}^2 - I^2\right)}$$

$$= \sqrt{(2.0 \times 10^{-3} \times 5.0 \times 10^{-6})(2^2 - 1^2)}$$

$$= 10^{-4}\sqrt{3}\ \text{C} = 1.732 \times 10^{-4}\ \text{C} = 1.732\ \mu\text{C}$$

14. Consider a small element of the loop of width dy and side a. The are of the element $dA = ady$. Since the area vector and the magnetic field vector point in the same direction, i.e.m the angle θ between the normal to the plane of the loop and the magnetic field is zero, the magnetic flux through the element is



$$d\phi = BdA \cos\theta = BdA \cos 0° = BdA$$

The total magnetic field lined with the loop is $\phi = \int BdA$

Since $B = B_0\left(\frac{y}{a}\right)\hat{k}$ depends upon y, we have

$$\phi = \int_{y}^{y+a} B_0\left(\frac{y}{a}\right) a\,dy$$

$$= B_0 \int_{y}^{y+a} y\,dy = \frac{B_0}{2}\,|y^2|_y^{y+a}$$

$$= \frac{B_0}{2}\,[(y+a)^2 - y^2] \quad \text{(i)}$$

Induced emf is $e = -\frac{d\phi}{dt} = \frac{B_0}{2}\frac{d}{dt}[(y+a)^2 - y^2]$

$$= -\frac{B_0}{2}\left[2(y+a)\frac{dy}{dt} - 2y\frac{dy}{dt}\right]$$

$$= -B_0 a\frac{dy}{dt}$$

The induced current is $I = \frac{|e|}{R} = \frac{B_0 a}{R}\frac{dy}{dt}$ (ii)

Since the magnetic field B points in the positive z-direction, it follows from Len's law that the direction of the induced current will be along the negative z-axis. Thus the current in the loop will flow in the counterclockwise direction.

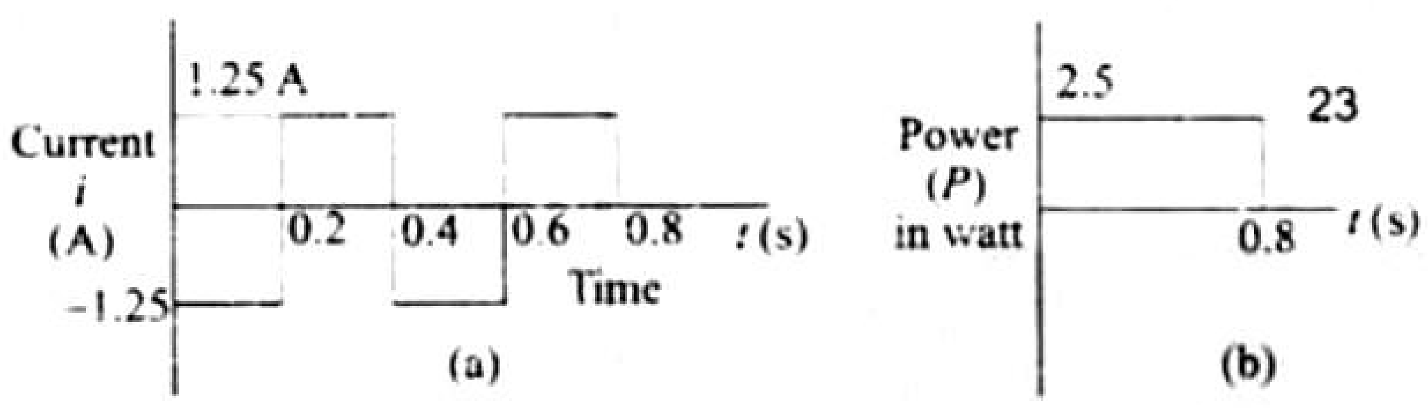
15. Induced emf

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA\cos 0°) = -NA\frac{dB}{dt}$$

$$= -100 \times 5 \times 10^{-3} \times \frac{0.8}{0.2} = -2\text{ V}$$

It is negative between 0 and 0.2 s when field is increased and positive between 0.2 and 0.4 s when field is decreased.

Current induced $i = \frac{\varepsilon}{R} = \frac{2}{1.6} = 1.25$ A



Power dissipated $= i^2R = (1.25)^2 \times 1.6 = 2.5$ W

Total energy supplied $= Pt = 12{,}000(2.5 \times 0.4) = 12 \times 10^3$ J

Now from the conservation of energy

$$12 \times 10^3 = m_1c_1\Delta\theta + m_2c_2\Delta\theta$$

$$\Rightarrow \quad = (m_1c_1 + m_2c_2)\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{12 \times 10^3}{m_1c_1 + m_2c_2}$$

$$= \frac{12 \times 10^3}{0.5 \times 4200 + 0.06 \times 500} = \frac{12 \times 10^3}{2130} = 5.6°$$

$$\Delta\theta = \theta_2 - \theta_1$$

$\Rightarrow$ Final temperature: $\theta_2 = \theta_1 + \Delta\theta = 30 + 5.6 = 35.6$°C

16. Let the current distribution in the circuit be as shown in the figure.

From KVL, $i = i_1 + i_2$ (i)

Applying KVL in loop $acdfa$, traversing the circuit clockwise, beginning at left corner,

$$E - L\frac{di_2}{dt} - i_2R_2 = 0 \quad \text{(ii)}$$

which on rearranging gives

$$\frac{dI_2}{E - I_2R_2} = \frac{1}{2}dt \quad \text{(iii)}$$

On integrating the equation (iii), we get

$$I_2 = \frac{E}{R_2}[1 - e^{-R_2t/L}]$$

Potential drop across inductor

$$|V_L| = L\frac{di_2}{dt} = Ee^{-R_2t/L}$$

Note that loop $acdfa$ is a simple RL circuit with time constant L/R_2. In the steady state, the inductor is short circuited, resistors R_1 and R_2 are in parallel arrangement.

So $i_1 = E/R_1$, $i_2 = E/R_2$.

On substituting numerical values, we have

$$\tau_L = \frac{L}{R_2} = \frac{0.4}{2} = 0.2\text{ s}$$

Steady-state current, $i_2 = \frac{E}{R_2} = \frac{12}{2} = 6$ A

$$I_2 = 6(1 - e^{-5t})$$

$$|V_L| = Ee^{-R_2t/2} = 12e^{-5t}$$

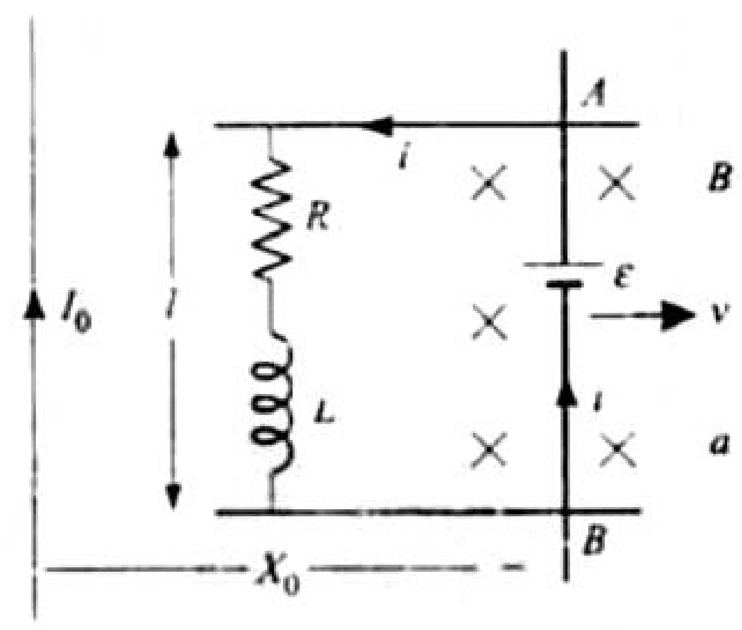
When the switch is opened the loop $abefa$ is an open circuit, current in R_2 is zero instantly while the loop $bcdeb$ the current decays to zero. Time constant of this loop is

$$\tau'_L = \frac{L}{R_1 + R_2} = \frac{0.4}{2+2} = 0.1\text{ s}$$

Current through R_1 at any time t is

$$i = i_0e^{-t/\tau'_L} = 6e^{-10t}$$

17. a. When bar AB slides away, the magnetic force acts on free electrons along negative Y-axis, so electrons move from end A to end B, making end A at positive potential relative to end B. So current in the circuit will flow from B to A as shown.



Let ε be the induced emf

According to Kirchhoff's second law,

$$-Ri - L\frac{di}{dt} + \varepsilon = 0 \quad \Rightarrow \quad \varepsilon = Ri + L\frac{di}{dt}$$

But $\varepsilon = \frac{d\phi}{dt}$ (numerically)

$$\therefore \quad \frac{d\phi}{dt} = Ri + L\frac{di}{dt} \quad \text{(i)}$$

This is the required relation.

b. Equation (i) can be expressed as

$$d\phi = Ridt + L\,di \quad \Rightarrow \quad d\phi = R\,dq + Ldi$$

Integrating, we get

$$\int_0^T d\phi = R\int_0^q dq + L\int_0^{i_1} di$$

$$[\phi]^T_{t=0} = Rq + Li_1$$

$\therefore$ Charge flown from $t = 0$ to $t = T$ will be

$$q = \frac{1}{R}[\phi(T) - \phi(0)] - \frac{Li_1}{R} \quad \text{(ii)}$$

Change in flux during the displacement of bar from $x = x_0$ to $x = 2x_0$ in time T is

$$\phi(T) - \phi(0) = \int_{x_0}^{2x_0} B dA = \int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x} l dx$$

$$= \frac{\mu_0 I_0 l}{2\pi} \log_e 2 \quad \text{(iii)}$$

$\therefore$ From Eq. (ii), charge flown

$$q = \frac{\mu_0 I_0 l}{2\pi R} \log_e 2 - \frac{Li_1}{R} \quad \text{(iv)}$$

c. The equation of decay of current in an LR circuit is given by

$$i = i_0 e^{-Rt/L} \quad \text{(v)}$$

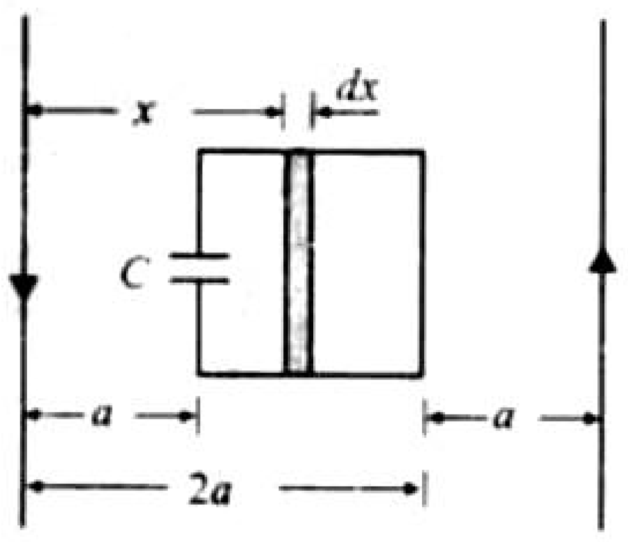
Given $i = \frac{i_1}{4}$, $i_0 = i_1$, $t = 2T - T = T$

$\therefore$ Substituting these values in Eq. (v), we get

$$\frac{i_1}{4} = i_1 e^{-RT/L} \Rightarrow e^{-RT/L} = \frac{1}{4}$$

or $\frac{RT}{L} = \log_e 4 \quad \therefore \frac{L}{R} = \frac{T}{\log_e 4}$

18. Let us first find the flux through the loop due to left wire.



$$\phi_L = \int_a^{2a} \frac{\mu_0 I}{2\pi x} a dx = \frac{\mu_0 I a}{2\pi}(\log_e 2)$$

This flux will be in outward direction because magnetic field due to wire is in outward direction and area vector is also in outward direction because anticlockwise direction is positive. Now flux due to right wire will also be same because loop is placed symmtrically w.r.t. both wires. So net flux through the loop is

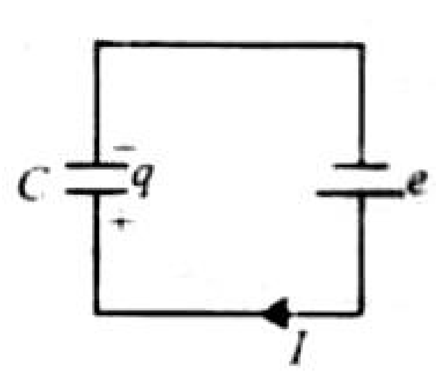
$$\phi = 2\phi_L = \frac{\mu_0 I a}{\pi} \log_e 2$$

Induced emf in the loop:

$$e = -\frac{d\phi}{dt} = -\frac{\mu_0 a(\log_e 2)}{\pi}\left(\frac{dI}{dt}\right)$$

$$= -\frac{\mu_0 I_0 \omega a}{\pi}(\log_e 2)\cos\omega t$$

Negative sign indicates that induced emf will be in clockwise sense because anticlockwise sense is positive. Finally, induced emf in the loop is $e = \frac{\mu_0 \omega a I_0(\log_e 2)}{\pi}\cos\omega t$ as shown in the figure.



Let q be the charge on the capacitor at any time, then

$$q = Ce = \frac{\mu_0 I_0 \omega a C \log_e 2 \cos\omega t}{\pi}$$

Current I in the circuit: $I = \frac{dq}{dt} = -\frac{\mu_0 I_0 \omega^2 a C \log_e 2}{\pi} \sin\omega t$

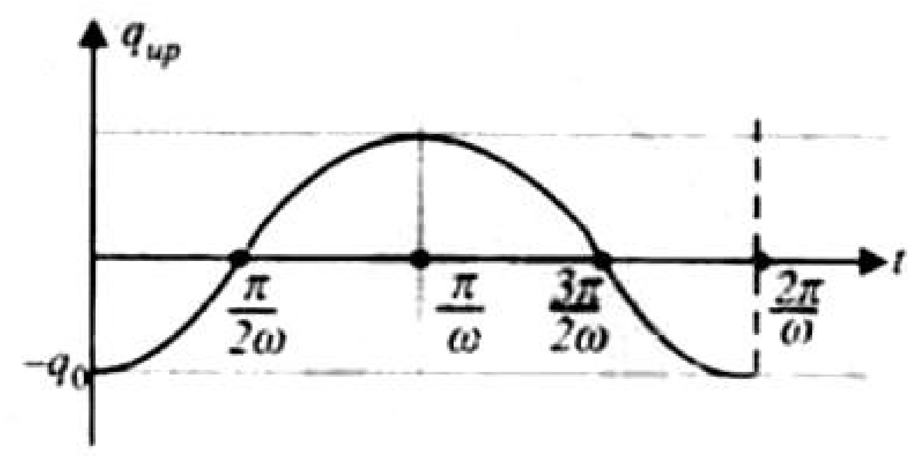
Hence, maximum current in the loop is

$$I_{max} = \frac{\mu_0 I_0 \omega^2 a C \log_e 2}{\pi}$$

Charge on the upper plate: $q_{up} = -q = -q_0 \cos\omega t$

where $q_0 = \frac{\mu_0 I_0 \omega a C(\log_e 2)}{\pi}$

The variation of q_{up} with time is as shown in the figure.



19. Inductive reactance

$$X_L = \omega L = (50)(2\pi)(35 \times 10^{-3}) = 11\ \Omega$$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\ \Omega$

Given, $V_{rms} = 220$V

Hence, amplitude of voltage

$$V_0 = \sqrt{2} V_{rms} = 220\sqrt{2}\ \text{V}$$

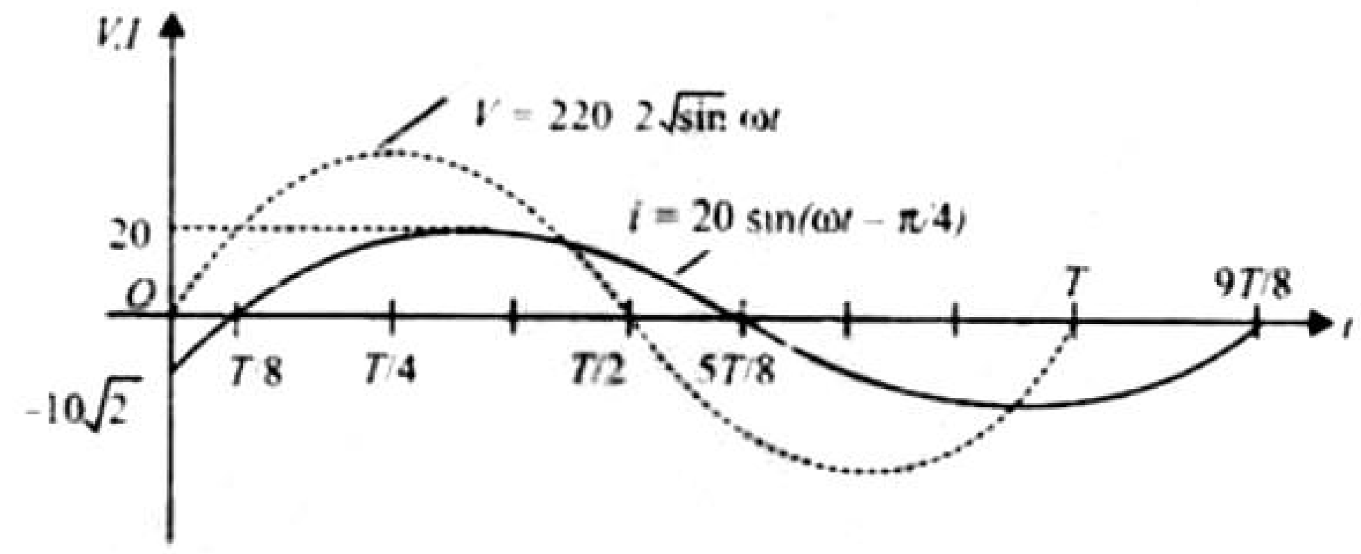
Amplitude of current $i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20$ A

Phase difference $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{11}{11}\right) = \frac{\pi}{4}$

In L-R circuit voltage leads the current. Hence, instantaneous current in the circuit is,

$$i = (20\ \text{A})\sin\left(\omega t - \frac{\pi}{4}\right)$$

Corresponding i–t graph is shown in the following figure.

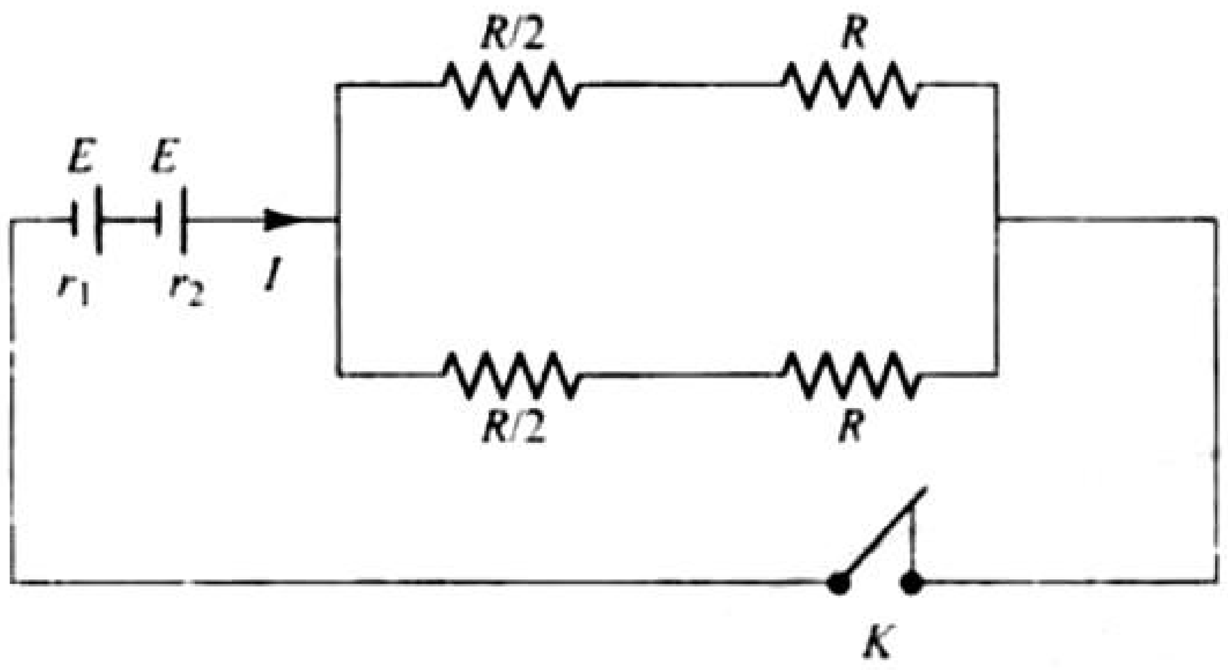


20. After a long time, steady state is reached in which impedance due to inductor (ωL for dc) is zero and that due to capacitance $\left(\frac{1}{\omega C}\right)$ becomes infinite, so equivalent circuit is shown in the figure.

Net external resistance $R_{ext} = \frac{\frac{R}{2} + R}{2} = \frac{3}{4}R$

Net internal resistance $R_{int} = r_1 + r_2$

$\therefore$ Current in circuit $I = \dfrac{2E}{\frac{3}{4}R + r_1 + r_2}$



The potential difference across the terminals of cell A is zero; so

$$E - Ir_1 = 0 \Rightarrow E - \frac{2Er_1}{\frac{3}{4}R + r_1 + r_2} \Rightarrow R = \frac{4}{3}(r_1 - r_2).$$

21. Inductive reactance

$$X_L = \omega L = (50)(2\pi)(35 \times 10^{-3}) \approx 11\ \Omega$$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(11)^2 + (11)^2} = 11\sqrt{2}\ \Omega$

Given $V_{rms} = 220$ V

Hence, impedance of voltage

$$V_0 = \sqrt{2}V_{rms} = 220\sqrt{2}\ \text{V}$$

$\therefore$ Amplitude of current $i_0 = \dfrac{V_0}{Z} = \dfrac{220\sqrt{2}}{11\sqrt{2}}$

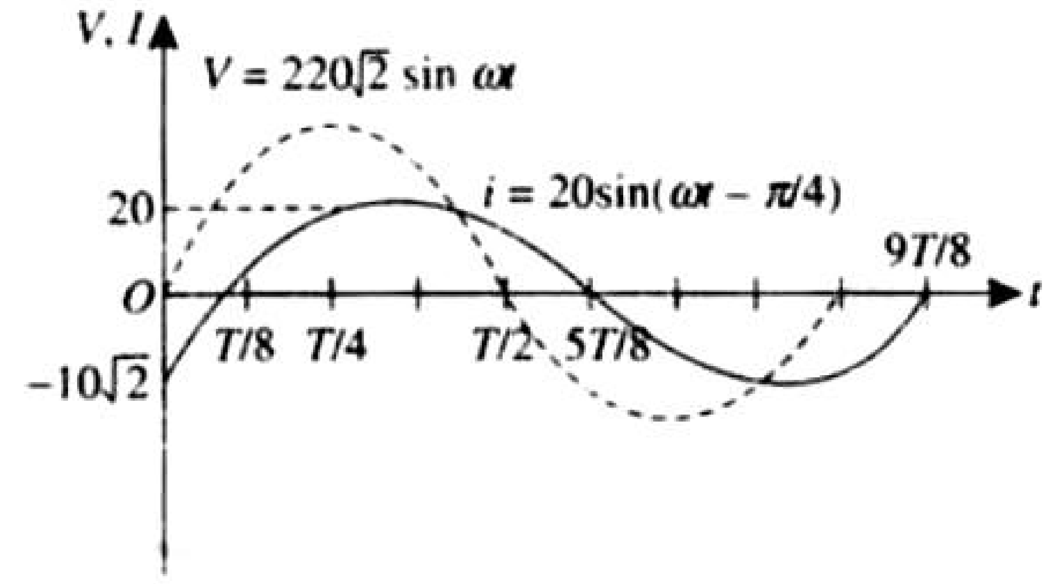
$$i_0 = 2\ \text{A}$$

Phase difference $\phi = \tan^{-1}\left(\dfrac{X_L}{R}\right) = \tan^{-1}\left(\dfrac{11}{11}\right) = \dfrac{\pi}{4}$

In L-R circuit voltage leads the current. Hence instantaneous current in the circuit is.

$$i = (20\ \text{A}) \sin(\omega t - \pi/4)$$

Corresponding i-t graph is shown in the figure.



22. Magnetic field b due to current in the solenoid is

$$B = \mu_0 ni = \mu_0 ni_0 \sin \omega t$$

Magnetic flux is given by

$$\phi = BA = (\mu_0 ni_0 \sin \omega t)(\pi a^2)$$

Induced emf in the cylindrical shell is

$$e = \frac{d\phi}{dt} = (\mu_0 ni_0 \omega \cos \omega t)(\pi a^2)$$

The resistance of the shell is

$$R = \rho\frac{\ell}{a} = \rho\frac{2\pi r}{Ld}$$

Finally, the induced current in the shell is

$$i = \frac{e}{R} = \frac{(\mu_0 ni_0 \omega \cos \omega t)(\pi a^2)}{2\pi r\rho / Ld}$$

$$= \left(\frac{\mu_0 ni_0 \omega a^2 Ld}{2\rho r}\right)\cos \omega t$$